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A Numerical Model of Wave Propagation on Mild Slopes

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ABSTRACT

Propagation of waves from Deep Ocean to a shoreline has been numerically modeled. Model equations govern combined effects of shoaling, refraction, diffraction and breaking. Linear, harmonic, and irrotational waves are considered, and the effects of currents and reflection on the wave propagation are assumed to be negligible. To describe the wave motion, mild slope equation has been decomposed into three equations that are solved in terms of wave height, wave approach angle and wave phase function. It is assumed that energy propagates along the wave crests, however, the wave phase function changes to handle any horizontal variation in the wave height. Model does not have the limitation that one coordinate should follow the dominant wave direction. Different wave approach angles can be investigated on the same computational grid. Finite difference approximations have been applied in the solution of governing equations. Model predictions are compared with the results of semicircular shoal tests performed by WHALIN (1971) and with the measurements of elliptic shoal experiment conducted by BERKOFF *et al.* (1982). Utility of the model to real coastal areas is shown by application to Obaköy on the Mediterranean Sea of Turkey.

ADDITIONALINDEX WORDS: *Wave propogation, deep ocean, shoaling, refraction*

INTRODUCTION

Precise numerical modeling of wave propagation from the deep ocean to a shoreline is quite important in coastal engineering. As waves travel over uneven topographies, they undergo a number of transformations. The wave ray method and linear gravity wave theory were used in the early works of wave transformation. Since the wave ray theory excludes wave diffraction, it is unable to predict the wave characteristics near coastal structures such as breakwaters. The mild slope equation which was proposed by BERKHOFF (1972) is applicable for computations of refraction and diffraction of linear waves. The mild slope equation is usually expressed in an elliptic form, and it turns to Helmholtz equation for uniform water depths. Since like the full linear wave equation, the mild slope equation is elliptic, it needs solution methods dealing with the whole region of interest in space. Comparisons with the solutions for the full linear equations by BOOIJ (1983) give confidence in the use of the mild slope equation for large slopes in suitable situations. To reduce the computational difficulties encountered in the solution of elliptic equation, parabolic approximation method was developed and used to study combined wave refraction and diffraction phenomena in coastal regions (LIU and TSAY, 1984; KIRBY and DARLYMPLE, 1983; RADDER, 1979). TANG and QUELLET (1997) applied parabolic approximation method to solve the proposed nonlinear equations of combined refraction diffraction problem. Two basic assumptions have been made in the parabolic approximation; one is that the wave field does not vary greatly from a unidirectional wave train, so waves have a principal propagation direction. The other assumption is that reflected waves are neglected. When the bottom contours are not straight and parallel as in the case of complex bathymetries, the requirement that one grid coordinate should follow the dominant wave direction causes problems (EBERSOLE, 1985).

In this paper, model equations similar to that proposed by EBERSOLE (1985) are solved numerically to deal with the combined refraction diffraction problem. The mild slope equation has been decomposed into three equations related to wave phase function,wave amplitude and wave approach angle that computes the wave field resulting from the transformation of an incident, linear wave as they propagate over irregular bottom configurations. Proposed model does not have the limitation that one coordinate should follow the dominant wave direction. Different wave approach angles can be investigated on the same computational grid. Finite difference approximations with variable mesh sizes are used to solve governing equations. Finer grid resolution can be generated in areas of complex bathymetry. The use of variable grid sizes considerably reduces the overall computational costs. Computationally, the numerical model is quite efficient for simulating wave propagation over large coastal areas subjected to varying wave conditions.

Model Equations

The complex velocity potential has been chosen as (EBERSOLE, 1985) ;

$$\phi = a e^{is} \tag{1}$$

in which, a: wave amplitude, s: scalar phase function of the wave.

If Eqn (1) is inserted to the equation that describes the propagation of harmonic linear waves in two horizontal dimensions, the following equation can be derived;

$$\frac{1}{a}\left[\frac{\partial^2 a}{\partial x^2}+\frac{\partial^2 a}{\partial y^2}+\frac{1}{CC_g}\left(\nabla a\cdot\nabla\left(CC_g\right)\right)\right]+k^2-\left|\nabla s\right|^2=0 \tag{2}$$

$$\nabla\left(a^2 CC_g \nabla s\right)=0 \tag{3}$$

in which —: horizontal gradient operator; C: wave celerity; Cg: group velocity; k: wave number calculated by the dispersion relation.

From vector analysis, the normal unit vector n to a scalar function is related to the normal vector N, which is found by taking the gradient of the function (DEAN and DALRYMPLE, 1991);

$$N = n\left|\nabla s\right| \tag{4}$$

The vector N points in the direction of the greatest change of phase function s, which is the wave propagation direction. The wave number vector is defined as;

$$\vec{k} = n\left|\nabla s\right| = \nabla s \tag{5}$$

It is clear that the wave number vector is nothing more than the wave number oriented in the wave direction.

To account the effect of diffraction, the wave phase function changes to consider any horizontal variation in the wave height. By the use of irrotationality of the gradient of the wave phase function following equations can be derived;

$$\nabla\times\left(\nabla s\right)=0 \tag{6}$$

$$\nabla s = \left|\nabla s\right|\cos(\theta)\vec{i}+\left|\nabla s\right|\sin(\theta)\vec{j} \tag{7}$$

in which , : unit vectors in the x and y directions, respectively; q(x,y): angle of incidence defined as the angle made between the bottom contour normal and the wave direction. q(x,y) can be found from the following expression;

$$\frac{\partial}{\partial x}\left(\left|\nabla s\right|\sin\theta\right)-\frac{\partial}{\partial y}\left(\left|\nabla s\right|\cos\theta\right)=0 \tag{8}$$

The following energy equation is used to determine wave amplitude;

$$\frac{\partial}{\partial x}\left(a^2 CC_g\left|\nabla s\right|\cos\theta\right)+\frac{\partial}{\partial y}\left(a^2 CC_g\left|\nabla s\right|\sin\theta\right)=0 \tag{9}$$

Eqn (6) together with Eqn (2) and Eqn (7) result in the set of three equations that will be solved in terms of three wave parameters, wave height H, local wave angle q and Ω—sΩ (EBERSOLE, 1985).

$$\left|\nabla s\right|=k^2+\frac{1}{H}\left[\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}+\frac{1}{CC_g}\left(\frac{\partial H}{\partial x}\frac{\partial CC_g}{\partial x}+\frac{\partial H}{\partial y}\frac{\partial CC_g}{\partial y}\right)\right] \tag{10}$$

$$\frac{\partial}{\partial x}\left(H^2 CC_g\left|\nabla s\right|\cos\theta\right)+\frac{\partial}{\partial y}\left(H^2 CC_g\left|\nabla s\right|\sin\theta\right)=0 \tag{11}$$

Eqns (8),(10) and (11) describe the refraction and diffraction phenomena. The basic assumptions are that the waves are linear, harmonic, irrotational, reflection is neglected and bottom slopes are small.

Numerical Solution

Solution method is a finite difference method that uses a mesh system in Cartesian coordinates. The finite difference approximations can handle the variations in the horizontal mesh sizes. The horizontal mesh size Δx in the x-coordinate is orthogonal to the horizontal mesh size Δy in the y-coordinate. The horizontal mesh sizes Δx and Δy can be different from each other. Also, Δx can vary along the x coordinate and Δy can vary along the y coordinate (BALAS and INAN, 2001a; BALAS and INAN, 2001b).

Input model parameters are the deep water wave parameters, wave height (H_0), wave approach angle (q_0) and the wave period (T). Partial derivatives in the x-direction are expressed by forward finite differences of order O(Δx), and the partial derivatives in the y-direction are expressed by central finite differences of order O(Δy2) in equation (8) and in Equation (11), whereas partial derivatives in the x-direction are approximated with backward finite differences of order O(Δx), and partial derivatives in the y-direction are expressed by central finite differences of order O(Δy2) in Equation (10). Wave breaking is controlled during the computations.

MODEL VERIFICATION

Semicircular Shoaling

Model predictions are compared with the results of a laboratory experiment (WHALIN, 1971). The wave tank used in the experiments is shown in Figure 1. Two different combinations of wave height and wave period have been simulated and compared with the experimental data. Along the lateral boundaries, the gradient of wave height perpendicular to side walls is assumed to be zero, and wave approach angles are assumed to be in the x direction. Topography is symmetric about y=3.048m. Water depth changes from 0.4572m to 0.1524m. Two different mesh sizes are used in the x-direction. The mesh size used is Δx=0.5 m and Δy=0.762m, in the x and y directions respectively. Lineer waves were produced at the water depth of 0.4572 m. On the slope, there are semicircular steps that result in strong wave convergence. Model predictions are compared with the measured data along the centerline of the wave tank for two different wave period and wave height combinations. Comparisons are shown in Figure 2 and in Figure 3 for a wave period of T=1.0 sec and wave amplitude of a=0.0195 m, and of T=2 sec and a=0.0075 m., respectively. Results of study performed by other researchers (LOZANO and LIU, 1980; LIU and TSAY, 1984; MADSEN and SORENSEN, 1992) are also presented in Figure 2 and in Figure 3 for comparison. Model simulation reflects well the effect of diffraction phenomenon and model predictions are in good agreement with the experimental results. On the semicircular steps, application of pure refraction theory results in crossing of wave rays and wave heights can not be computed. Consideration of diffraction prevents this phenomenon. Therefore simulation of this experiment is a good indicator of the model to predict the effects of diffraction.

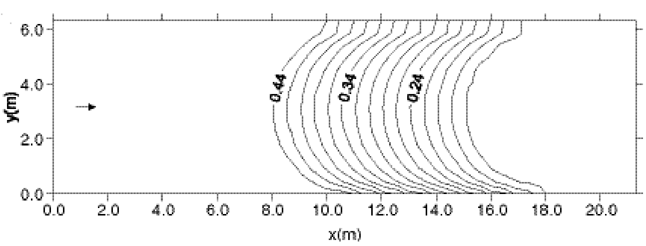

Figure 1. The bathymetry of wave tank (water depths are in m) (WHALIN, 1971).

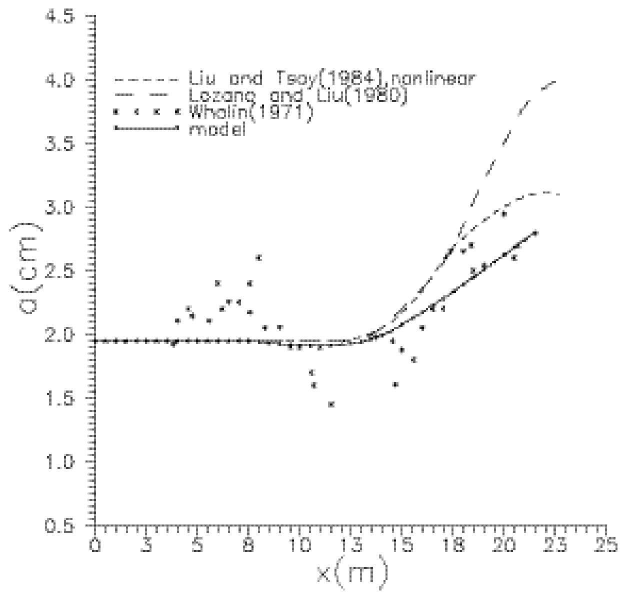

Figure 2. Comparison of model predictions (T=1s, a=0.0195m and q=00).

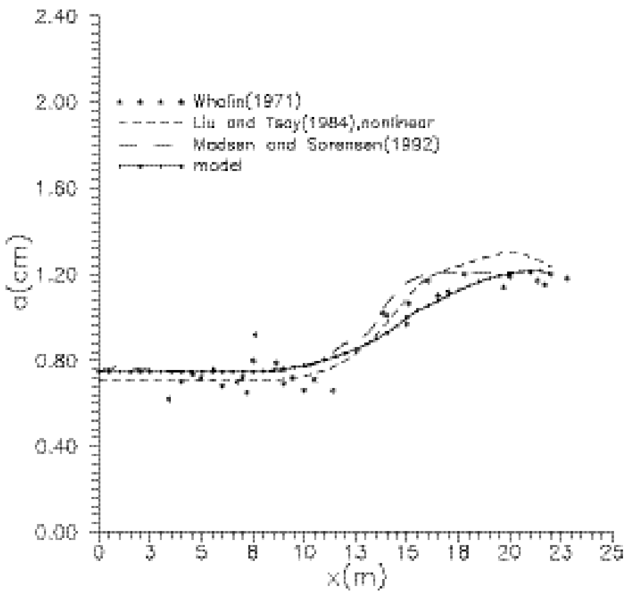

Figure 3. Comparison of model predictions (T=2s, a=0.0075m and q=00).

Elliptical Shoaling

In the second application, model predictions are compared with the results of wave tank experiment done by BERKHOFF *et al*. (1982). The wave period of incoming waves is T=1s, and the wave height is H=0.01058 m. Wave approach angle is 18.5°. Water depths in the tank decreases from 0.45m with a bottom slope of 1/50. The bathymetry of the wave tank is given in Figure (4).

In the numerical model grid sizes are selected as Δx=0.5m and Δy=0.5m. Numerical model predictions along the cross section of x=15 m, x=19 m and y=8 m are compared with the experimental data of BERKHOFF *et al*.(1982), and presented in Figure (5), in Figure (6) and in Figure (7) respectively. For comparison, numerical predictions of KIRBY and DALRYMPLE (1984) are depicted in the figures as well. Model predictions are in good agreement with the measurements. Model well reflects the experimental results near the shoal area.

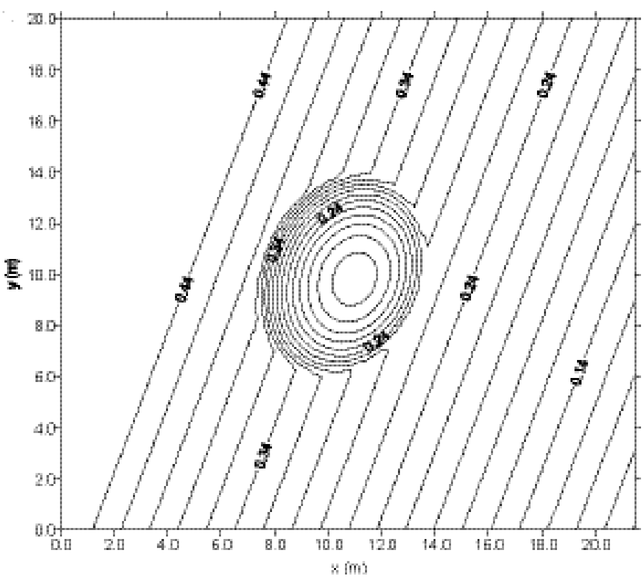

Figure 4. Wave tank bathymetry (water depths are in m) (BERKOFF *et al*., 1982).

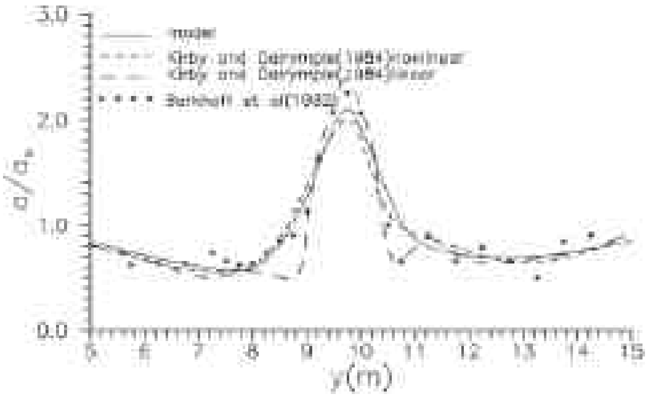

Figure 5. Variation of relative wave height at x=15m.

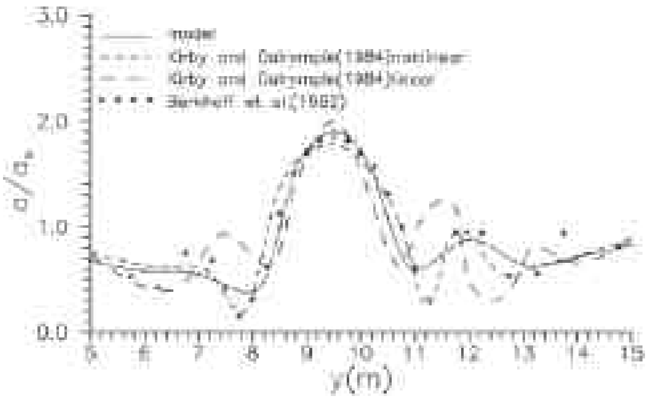

Figure 6. Variation of relative wave height at x=19m.

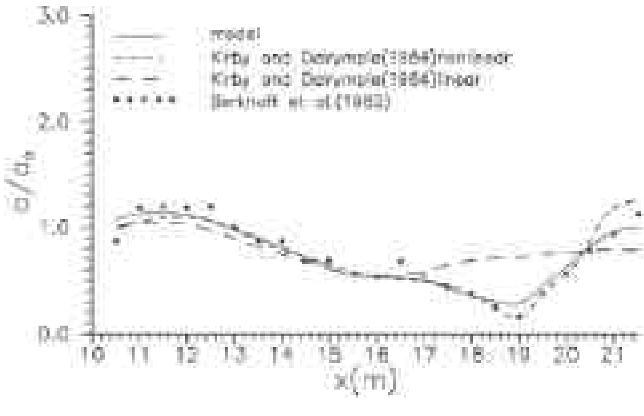

Figure 7. Variation of relative wave height at y=8m.

Model Application to Obaköy

Model has been applied to Obaköy which is located at the Mediterranean Sea coast of Turkey. In the coastal waters of Obaköy a sea outfall construction has been planned by the authorities. The map of the coastal area is given in Figure (8). The bathymetry for the area is shown in Figure (9). For the area, wave transformations from the dominant wave directions, which are the S and SW directions, are simulated. The deep water wave parameters are used to specify the offshore boundary conditions and zero gradient boundary conditions are applied for wave heights and wave angles along the lateral boundaries. Deep water parameters are selected as, wave period T=10.3 s, wave height H=6.5 m. Model predictions are presented in Figure (10) and in Figure (11) for waves approaching from S and SW directions respectively. Model provides reasonable estimations for the area. Waves converge on the shoal, conveyance of energy onto shoal results in the decrease of wave heights. Model can be used successfully for the areas having complicated bathymetries.

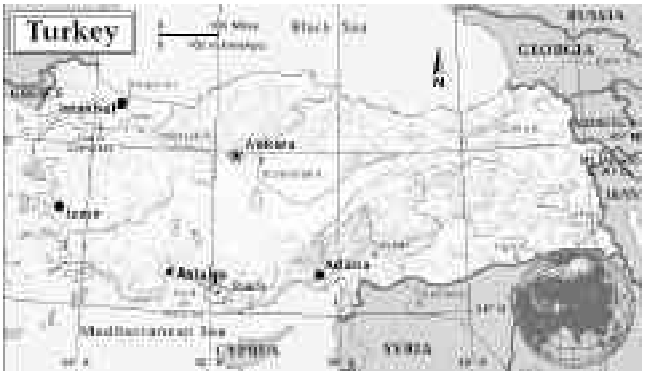

Figure 8. Map of the coastal area.

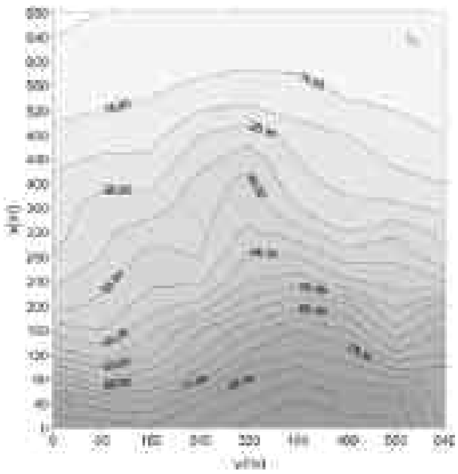

Figure 9. Bathymetry of the computational area (water depths are in m).

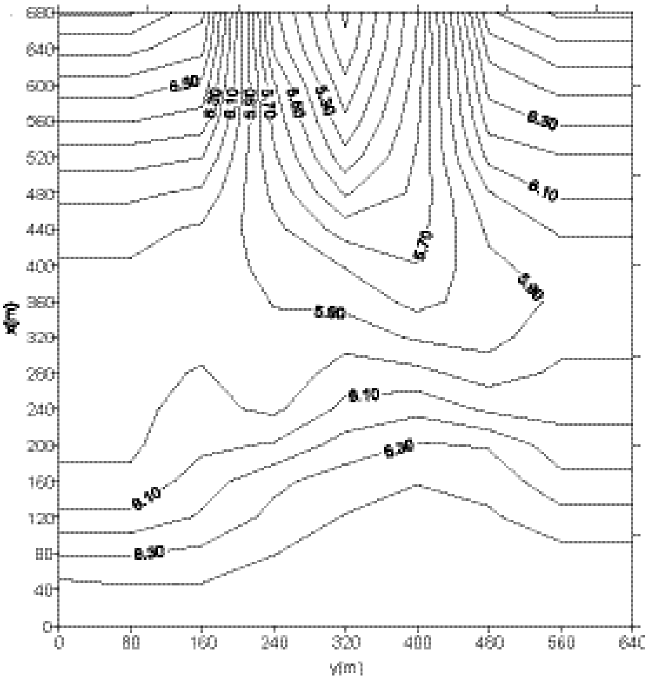

Figure 10. Wave heights(m) in the computational area for waves approaching from S direction .

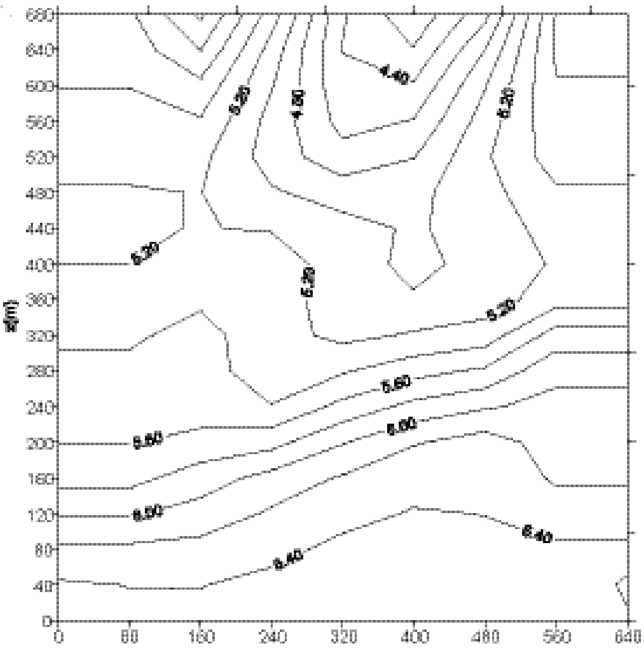

Figure 11. Wave heights(m) in the computational area for waves approaching from SW direction .

For waves approaching from S direction, waves are refracted onto the shoals causing conveyance of energy and that results in decrease of wave heights just outside the shoals. For wave approaching from SW direction, waves converge on the nearest shoal but also they refract around to the far shoal. Less energy is propagated towards the shore causing rather smaller wave heights.

CONCLUSIONS

A numerical model has been developed to simulate the wave transformation of monochromatic linear waves as they propagate over irregular bathymetries. Model can simulate the effect of pure refraction or effect of refraction together with diffraction which is important over complex bathymetries. Model overcomes the limitation of the parabolic approximation that one grid coordinate should follow the dominant wave direction. There is no assumption regarding the curvature of the wave height in any direction. Only one computational domain is enough to simulate the transformation of waves from different directions with different approach angles. The possibility of selection of variable grid size allows to fit finer or coarser meshes to the solution domain. Therefore computationally, the numerical model is quite efficient for simulating wave propagation over large coastal areas subjected to varying wave conditions.

Model predictions are compared with the experimental results. The agreement between the model predictions and the experimental results is highly encouraging. Model successfull application to a real coastal water body has been demonstrated. Developed model is a reliable tool for simulating the transformation of linear waves over complicated bathymetries.

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