

# Plant Identification Through Images: Using Feature Extraction of Key Points on Leaf Contours

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APPLICATION ARTICLE

## Plant identification through images: Using feature extraction of key points on leaf contours<sup>1</sup>

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- *Premise of the study:* Because plant identification demands extensive knowledge and complex terminologies, even professional botanists require significant time in the field for mastery of the subject. As plant leaves are normally regarded as possessing useful characteristics for species identification, leaf recognition through images can be considered an important research issue for plant recognition.
- *Methods:* This study proposes a feature extraction method for leaf contours, which describes the lines between the centroid and each contour point on an image. A length histogram is created to represent the distribution of distances in the leaf contour. Thereafter, a classifier is applied from a statistical model to calculate the matching score of the template and query leaf.
- *Results:* The experimental results show that the top value achieves 92.7% and the first two values can achieve 97.3%. In the scale invariance test, those 45 correlation coefficients fall between the minimal value of 0.98611 and the maximal value of 0.99992. Like the scale invariance test, the rotation invariance test performed 45 comparison sets. The correlation coefficients range between 0.98071 and 0.99988.
- *Discussion:* This study shows that the extracted features from leaf images are invariant to scale and rotation because those features are close to positive correlation in terms of coefficient correlation. Moreover, the experimental results indicated that the proposed method outperforms two other methods, Zernike moments and curvature scale space.

Key words: classifier of statistical model; edge detection; feature extraction; leaf recognition.

Because plant identification demands extensive knowledge and uses complex terminology, even professional botanists need to take much time in the field to master plant identification (Rademaker, 2000). Plant identification by information systems has often been regarded as a possibility. By employing personal digital devices to photograph the whole plant or a portion of the plant, information systems can be used to perform plant recognition. Plants may be recognized through the leaves, flowers, roots, and fruits, which reflect the diversity of plant shapes available within an organism. In particular, the shape of leaves and the floral organs-the modified leaves-are especially important (Tsukaya, 2006), with the leaves considered an especially useful characteristic for species identification (Gu et al., 2005; Du et al., 2007; Wu et al., 2007). For example, the free mobile app Leafsnap (http://leafsnap.com) has been developed to identify tree species from photographs of their leaves. Marcysiak (2012) examined the morphology of Salix herbacea L. leaves for intraspecific morphological variation. A total of 3890 leaves from 503 individuals were statistically analyzed based on leaf shape characters. A notable variation of shape characters of leaves of S. herbacea was found on different levels, including intra- and interindividual samples. For example, Gailing et al. (2012) identified morphological species and differentiation

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patterns on two species, *Q. rubra* L. and *Q. ellipsoidalis* E. J. Hill, which hybridize with each other. The two plant species were identified as two clusters when leaf morphological characters were measured. Furthermore, two populations of *Q. ellipsoidalis* were differentiated from eight other populations through analysis of leaf morphological characters. Therefore, leaf recognition through images can be considered an important research issue for plant recognition.

Shape is one of the most important features for describing an object. Humans can easily identify various objects and classify them into different categories solely from the outline of an object. Shape often carries several types of contour information, which are used as distinctive features for the classification of an object. In the MPEG-7 standard, shape descriptors can be divided into region-based shape descriptors and contour-based shape descriptors (Zhang and Lu, 2003a). Region-based shape descriptors such as Zernike moments (Wee and Paramesran, 2007) describe a shape based on both boundary and interior pixel information. Region-based shape descriptors can be used to depict several complex objects with filled regions (Bober et al., 2002), and can capture both the interior contents and boundary information of an object in an image. However, contourbased descriptors only exploit the boundary information of an object, and include the conventional representation and structural representation. Conventional descriptors such as curvature scale space (CSS) (Mokhtarian et al., 2005) retain the overall shape of an object during calculation. Structural descriptors such as chain code fragment the shape of an object into different boundary segments (Zhang and Lu, 2003b).

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Because the morphology of leaves is commonly used for plant identification, the studies shown in Table 1 have examined the shape and morphological description for plant leaves. As leaf recognition can be regarded as an image classification issue, various types of neural networks were proposed for identifying the species to which a given leaf belongs. Chaki and Parekh (2011) presented a schematic for the automated detection of three classes in a plant species by analyzing the shapes of leaves and using several neural network classifiers. Gao et al. (2010a) proposed a neural network classifier based on prior evolution and iterative approximation for leaf recognition. Huang and He (2008) applied probabilistic neural networks for the recognition of 30 types of broad-leaved trees. Furthermore, Wu et al. (2007) also introduced the probabilistic neural network to classify 32 types of plants. Other various classification methods were proposed for leaf recognition in addition to neural networks. Ehsanirad (2010) trained a classifier to categorize 13 types of plants with 65 new or deformed leaves during the testing process. In the Du et al. (2007) study, a moving mediancentered hypersphere classifier was adapted to perform the classification. Hajjdiab and Al Maskari (2011) presented an approach for identifying leaf images based on the cross-correlation of distances from the centroid to the leaf contour.

Feature extraction for leaf images requires consideration of which features are most useful for representing the leaves and which methods can effectively code leaf morphologies (Wu et al., 2006). A leaf of a given species normally represents a specific shape or contour; therefore, this characteristic is a reliable and meaningful indicator for leaf representation. The main contribution of this study is to propose a feature extraction method for leaf contours that describes these significant turning points. Moreover, a classifier of a statistical model is proposed for similarity matching with different numbers of features.

#### MATERIALS AND METHODS

*Leaf recognition framework*—The leaf recognition framework was divided into leaf modeling and leaf recognition. For leaf modeling, leaves belonging to the same species were used to detect and extract leaf features. The extracted features were then used for leaf modeling, creating a leaf model for each leaf species in the database. During leaf recognition, a query leaf was also tested by detecting feature points and feature extraction. Using these features, the recognition system can identify the best matching model and recognize the species of the query leaf.

**Object contour**—The contour of object O in image I can be detected to generate the set  $\xi$ , which collects all contour points p in a Cartesian coordinate system. These contour points can be used to calculate the centroid C of the object using Equation 1.

$$C = \frac{\sum_{p \in \xi} p}{|\xi|},\tag{1}$$

where  $|\xi|$  represents the number of edge points in set  $\xi$ . All contour points are collected in a clockwise order and stored in set  $\xi$ . As several segments of an object contour contain redundant points, these redundant points can be removed through sampling. The sampling process is to select the contour points from every five points in the set  $\xi$ . Thereafter, the selected points are stored in another set *S*. Figure 1 illustrates the process of detecting contour points. The contour points of the leaf in Fig. 1A are sampled to result in Fig. 1B.

*Feature extraction*—In the object contour, straight lines are created between centroid *C* and each contour point  $\rho$ . Thereafter, the lengths of the straight lines can be calculated. Suppose that a set of contour points is  $S = \{\rho_1, \rho_2, ..., \rho_n\}$ . The line length *len<sub>i</sub>* can be computed as

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TABLE 1. Methods and features used in leaf recognition studies.

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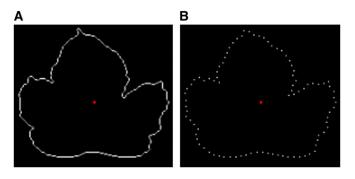


Fig. 1. Detection of contour points. (A) Contour and centroid C of leaf. (B) Sampling result of contour points.

$$len_i = |C\rho_i| \quad \forall \rho_i \in S \tag{2}$$

The distance features are normalized to create a histogram that represents the distribution of distances in the object contour. All  $Len_i$  are divided by the greatest  $Len_{max}$  and collected in *R* to normalize the length features.

$$R = \left\{ r_i \mid r_i = Len_i / Len_{\max} \right\}$$
(3)

Intradifference, the difference in a leaf species at individual leaves, may cause mistaken recognition. To deal with the intradifference problem and make the classification stable, the proposed feature is processed through the fuzzy logic method. The degrees of probability from probabilistic logic (Lukasiewicz and Straccia, 2009) is introduced into the histogram, where the frequency of each bin is replaced by fuzzy scores. The fuzzy score algorithm transforms the normalized features into fuzzy scores as shown in the algorithm in Appendix 1. For example, the feature value of A is 4.25 and it is transformed into two fuzzy values [0.5, 0.5]. The two fuzzy values are accumulated into bins [3,4] and [4,5] in the histogram. For point B, three fuzzy values are [0,1,0] for bins [3,4], [4,5], and [5,6]. Two fuzzy values of point C are [0.3, 0.7] for bins [4,5] and [5,6]. Figure 2 shows that three feature values are transformed into *N* classes, which is set as N = 24 in this study. The *j* represents an array and  $r_i$  is assigned to the given class based on the following rules  $v[\bullet]$ :

$$\begin{cases} v[0] = v[0] + 1, \text{ if } 0 \le r_i < \frac{1}{2N} \\ v[j-1] = v[j-1] + \left(\frac{2j+1}{2} - r_i \times N\right) \\ v[j] = v[j] + \left(r_i \times N - \frac{2j-1}{2}\right) \end{cases}, \text{ if } r_i \le \frac{2j+1}{2N}, j \in [1, \dots, N-1] \\ v[j] = v[j] + \left(\frac{2j+3}{2} - r_i \times N\right) \\ v[j+1] = v[j+1] + \left(r_i \times N - \frac{2j+1}{2}\right) \end{cases}, \text{ if } r_i > \frac{2j+1}{2N}, j \in [1, \dots, N-1] \\ v[N-1] = v[N-1] + 1, \text{ if } 1 - \frac{1}{2N} \le r_i \le 1 \end{cases}$$

$$(4)$$

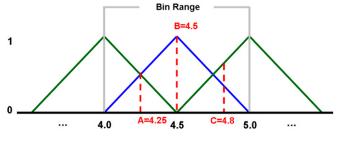


Fig. 2. Probabilistic logic diagram.

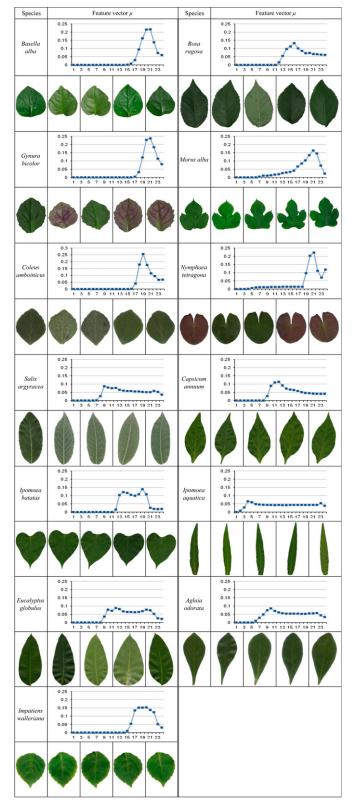


Fig. 3. Thirteen species of plant leaves collected for this study, including sample leaves and feature histograms.

TABLE 2.	Recognition results of the	e proposed features	for the training set and the test set.

Species	Training set	Testing set
Basella alba	(18, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(15, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Rosa rugosa	(18, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(19, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Gynura bicolor	(16, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(14, 3, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Morus alba	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
Coleus amboinicus	(19, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
Nymphaea tetragona	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
Salix argyracea	(16, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(19, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Capsicum annuum	(19, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(16, 0, 3, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Ipomoea batatas	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
Ipomoea aquatica	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
Eucalyptus globulus	(18, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
Aglaia odorata	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
Impatiens walleriana	(18, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(18, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Total	(242, 16, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(241, 12, 5, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Total (%)	(93.1, 6.1, 0.8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(92.7, 4.6, 1.9, 0.8, 0, 0, 0, 0, 0, 0, 0, 0, 0)

Each object can result in a histogram that represents information regarding the contour. Therefore, these resulting histograms can be used to estimate the matching degree between any two objects.

*Classifier of statistical model*—Once the leaf features  $X = (x_1, x_2, ..., x_n)$  are extracted from a leaf, the leaf classifier can be expressed using the following equation:

$$\hat{i} = \arg\max_{T_i} P(T_i \mid X)$$
(5)

where  $T_i$  is the model of leaf *i* and  $P(T_i | X)$  is the discriminant function of  $T_i$ . Bayes' theorem indicates

$$P(T_i | X) = \frac{f(X | T_i) \times P(T_i)}{f(X)}$$
(6)

where  $f(\cdot)$  is the probability density function. The f(X) is the common term for identifying the maximum probability because  $\hat{i}$  is estimated. If we assume a uniform prior probability  $P(T_i)$  on the species identity, the discriminant function in Equation 5 can be simplified as

$$\hat{i} = \arg\max_{T_i} f(X | T_i)$$
(7)

To reduce computational complexity, we further assume that  $x_1, x_2, ..., x_n$  are mutually independent features. Equation 3 can be transformed into Equation 4

$$\hat{i} = \arg\max_{T_i} \prod_{j=1}^n f(x_j \mid T_i)$$
(8)

If x is distributed normally with mean  $\mu$  and variance  $\sigma^2$ , then  $f(x) \sim N(\mu, \sigma^2)$ 

$$f(X|T_i) = \prod_{j=1}^n \frac{1}{\sqrt{2\pi\sigma_i^j}} \exp\left(-\frac{\left(x_j - \mu_i^j\right)^2}{2\sigma_i^j}\right)$$
(9)

To compute the exponential value efficiently, we use the logarithm of the discriminant function

$$Log(f(X|T_{i})) = -\frac{1}{2} \sum_{j=1}^{n} \left( \frac{(x_{j} - \mu_{i}^{j})^{2}}{\sigma_{i}^{j}} + Log(2\pi\sigma_{i}^{j}) \right)$$
(10)

which is referred to as score function. Thereafter, *c* sample leaves of each species in the training set are used to estimate the parameters  $\mu_i^j$  and  $\sigma_i^j$  of each  $T_i$  as follows:

$$\mu_i^j = \frac{1}{c} \sum_{k=1}^c x_i^{jk}$$
(11)

TABLE 3. Recognition results of Zernike moments for the training set and the test set.

Species	Training set	Testing set
Basella alba	(14, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(9, 3, 0, 0, 2, 0, 0, 0, 1, 0, 0, 0, 0)
Rosa rugosa	(10, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(8, 4, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0)
Gynura bicolor	(10, 4, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(5, 3, 1, 2, 3, 1, 0, 0, 0, 0, 0, 0, 0)
Morus alba	(14, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(10, 3, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Coleus amboinicus	(13, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(11, 3, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Nymphaea tetragona	(14, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0)	(14, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Salix argyracea	(16, 3, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(3, 8, 1, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Capsicum annuum	(10, 3, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(10, 2, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0)
Ipomoea batatas	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Ipomoea aquatica	(13, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(12, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Eucalyptus globulus	(11, 1, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(11, 3, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Aglaia odorata	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Impatiens walleriana	(14, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(14, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Total	(159, 22, 11, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0)	(137, 33, 7, 6, 8, 1, 1, 1, 1, 0, 0, 0, 0)
Total (%)	(81.5, 11.3, 5.6, 1, 0.5, 0, 0, 0, 0, 0, 0, 0, 0)	(70.3, 16.9, 3.6, 3.1, 4.1, 0.5, 0.5, 0.5, 0.5, 0, 0, 0, 0)

TABLE 4.	Recognition results of curvature scale sp	bace for the training set and the test set.

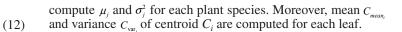
Species	Training set	Testing set
Basella alba	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Rosa rugosa	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(14, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Gynura bicolor	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(10, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Morus alba	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Coleus amboinicus	(14, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Nymphaea tetragona	(13, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(13, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Salix argyracea	(13, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(9, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Capsicum annuum	(11, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(6, 7, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Ipomoea batatas	(14, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(11, 2, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Îpomoea aquatica	(14, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(13, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Eucalyptus globulus	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Aglaia odorata	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Impatiens walleriana	(15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(12, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Total	(184, 11, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(163, 26, 5, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)
Total (%)	(94.4, 5.6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(83.6, 13.3, 2.6, 0.5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

$$\sigma_{i}^{j} = \frac{1}{c} \sum_{k=1}^{c} \left( x_{i}^{j,k} - \mu_{i}^{j} \right)^{2}$$

where  $x_i^{j,k}$  represents the k-th feature of the j-th sample leaves in the i-th species.

#### **RESULTS AND DISCUSSION**

This study examined 13 species of fresh plant leaves as shown in Fig. 3. This figure also includes some sample leaves and the feature histogram of a given leaf. For each species, separate images of 40 plant leaves were used to evaluate the proposed features and algorithms. The first 20 images in each species are regarded as the training set and the last 20 images are the test set. Furthermore, a feature histogram v[•] was created for all leaves. Equation 11 and Equation 12 are applied to



$$C_{mean_i} = \frac{\sum_{T_i} C_i}{|T_i|} \tag{13}$$

$$C_{\text{var}_{j}} = \frac{\sum_{i} \left(C_{i} - C_{mean_{i}}\right)^{2}}{\left|T_{i}\right|}$$
(14)

Table 2 shows that the recognition results for the training set and test set are indicated as a tredecuple ordered list of correct representatives. The ordered list reports the result of the recognition

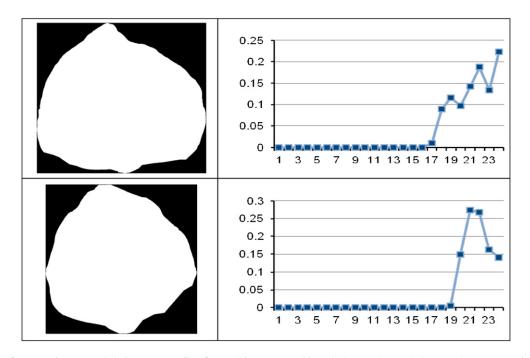


Fig. 4. Two leaf contour images and their corresponding feature histograms. Although the two leaves belong to the same species, their histograms present two greatly different feature curves.

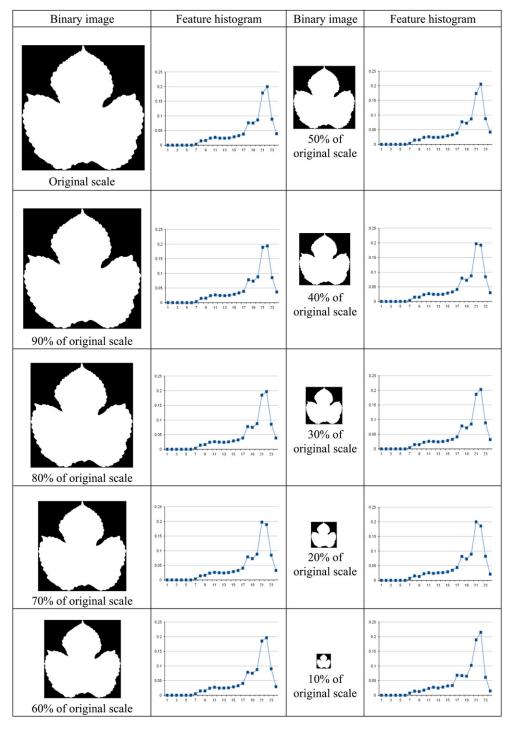


Fig. 5. A binary leaf image presented at sizes from 90% to 10% of the original size to verify scale invariance, with their corresponding histograms. In these histograms, the horizontal axis and vertical axis represent feature number and feature value, respectively.

results where the first position is the correct identification of the plant species. The listed second position is the recognition result identifying the plant species as the second probable plant species. It is expected that the correct representative should be ranked as high as possible. The results in Table 2 show that the top value of the tredecuple reaches 93.1% and the first two can even achieve 99.2% for the training set. In comparison with the test set, the top value achieves 92.7% and the first two values

can achieve 97.3%. The recognition performances for the training set and test set are substantially close.

Zernike moments and curvature scale space are two popular methods that are both invariant to scale and rotation and were tested in the same experimental setup. The Zernike moments derive from a set of complex polynomials orthogonal over the interior of a unit circle and defined in the polar coordinates. The recognition results of the two methods for the

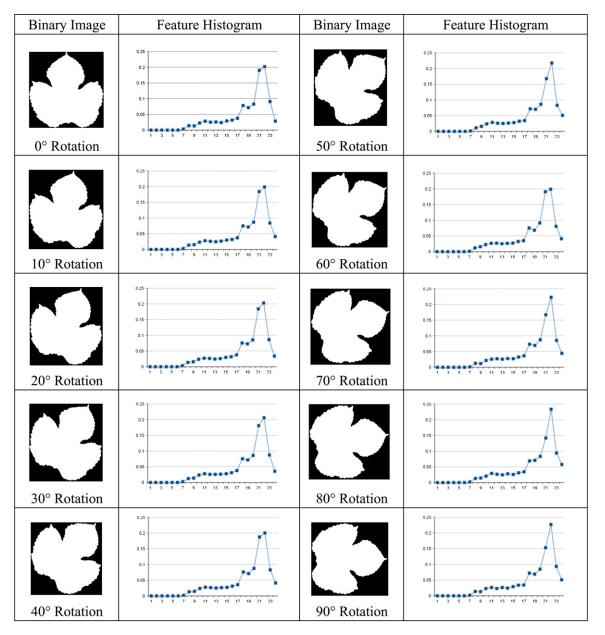


Fig. 6. A binary leaf image rotated clockwise from  $10^{\circ}$  to  $90^{\circ}$  to verify rotation invariance, with corresponding histograms. In these histograms, the horizontal axis and vertical axis represent feature number and feature value, respectively.

training set and test set are shown on Table 3 and Table 4. If we compare the recognition rate for the first probable plant species, the results shown in Tables 2–4 indicate that the proposed method outperforms Zernike moments and curvature scale space.

Numerous leaves belonging to the same species may still possess great differences in contour. For example, Fig. 4 shows two leaf contours and their corresponding feature histograms. Although the two leaves belong to the same species, their histograms present two greatly different feature curves. An erroneous recognition happens when the feature curve of a given leaf is closer to the model of another species than that of the correct species. The problem would be solved by building multiple models for the same species, which is a potential research issue for other researchers to investigate. The experimental results indicate that the correct recognition rate is 92.7% if we strictly examine the first-position plant of the recognition result. In other words, the erroneous recognition rate is approximately 7.3%. The cause of the erroneous recognition may involve the use of the parameter N in Equation 4, which in feature extraction may affect the fuzzy feature. When N is set higher, the leaves belonging to the same species are regarded as different species. When N is set lower, the leaves belonging to the different species are seen as same species. The parameter determination issue is also similar to the length of an interval for sampling contour points.

*Scale invariance*—To verify the scale invariance, a binary image was shrunk to various sizes from the original image (from 90% to 10%). The features of the different-sized images

were extracted to create their corresponding histograms as shown in Fig. 5. Correlation coefficients were computed for the similarity of any two scale ratios. This test was performed on 45 comparison sets. These 45 correlation coefficients fell between the minimal value 0.98611 and the maximal value 0.99992, indicating a strongly positive correlation. The results indicate the 10 feature histograms are very similar in terms of correlation coefficients. The curve in the feature histogram does not fluctuate considerably even when the image is shrunk to 10% of the original scale. These results also confirm that the proposed features are invariant to scale.

**Rotation invariance**—To verify the rotation invariance, a binary image was rotated clockwise to various degrees from the original degree (from  $10^{\circ}$  to  $90^{\circ}$ ). The features of the rotated images were extracted to create their corresponding histograms as shown in Fig. 6. Like the scale invariance test, the rotation invariance test was performed for 45 comparison sets using correlation analysis. The range of the correlation coefficients was between 0.98071 and 0.99988. These results indicate that the curves of these histograms have a very similar appearance, indicating the property of rotation invariance in the proposed features.

#### CONCLUSIONS

This study presents a feature extraction method for shape description and a classifier of a statistical model for different feature dimensions. The extracted features are invariant to scale and rotation, and the proposed method outperforms Zernike moments and curvature scale space. If the shape of leaves within a species varies substantially, multiple leaf templates are suggested for creating the species leaf model. We will extract more features from the patterns of the leaf vein and positions of the petioles of leaves in a future study to improve recognition performance.

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APPENDIX 1. The fuzzy score algorithm.

#### Begin

Create an *N*-dimensional matrix  $v[\bullet]$ 

Move the cursor to the first  $r_i$  in R

Run the following step until the cursor moves to the last one.

#### Begin

 $iBin = Floor(r_i \times N)$ If  $r_i < \frac{1}{2N}$  Then v[0] = v[0] + 1ElseIf  $r_i \ge 1 - \frac{1}{2N}$  Then v[N-1] = v[n-1] + 1

Else

Begin

Mid=(iBin+0.5)/N

If  $r_i \leq Mid$  Then

Begin

$$v[iBin-1] = v[iBin-1] + (Mid - r_i) \times N$$
$$v[iBin] = v[iBin] + \left(r_i - Mid + \frac{1}{N}\right) \times N$$
End

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Else

### **Begin** $v[iBin] = v[iBin] + \left(Mid + \frac{1}{N} - r_i\right) \times N$

 $v[iBin+1] = v[iBin+1] + (r_i - Mid) \times N$ 

End

End

Move the cursor to the next  $r_i$ 

End End