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Open Channel Flow Friction Factor: Logarithmic Law

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ABSTRACT

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The friction factor in open channel flow is considered to increase proportionally with wall roughness compared to that of pipe flow because of undulation at the free surface. The undulation is considered to be driven from the bottom and side walls. The increment factor associated with free surface undulation, shape distortion, and meandering is related to the Manning coefficient, and the incremented friction factor is substituted into the friction factor of circular pipe smooth (stretching) turbulent flow so as to yield the friction factor for open channel flow. The new equation for friction factor or mean velocity provides similar results to the Manning equation and the Ganguillet-Kutter (GK) equation for small streams. However, it is found that the Manning equation results in larger values of mean velocity in wide rivers compared with the GK equation and the new logarithmic equation based on the concept termed as "smooth turbulent," which appears inappropriate in describing the turbulent flow mechanism. Smooth turbulent may have to be denoted as "stretching turbulent" because of stretching of the velocity profile as the Reynolds number increases. Based on the comparison results with the other two empirical equations, the new logarithmic equation could be suitably adopted in open channel flows.

ADDITIONAL INDEX WORDS: *Logarithmic equation, increment factor, frictional head loss, stretching turbulent flow, free surface undulation.*

INTRODUCTION

Resistance in open channel flows is caused by various factors. Rouse (1965) suggested four factors for incurring flow resistance in open channel flow: (1) skin friction, (2) surface distortion, (3) form drag, and (4) local acceleration. When a channel is straight and uniform and its wall has a fixed and immobile roughness, the last two factors do not act as sources of flow resistance. In this case, skin friction may occur in both pipe flows and open channel flows, but surface distortion occurs only in open channel flows. Thus, surface distortion (undulation) might be one of the most important features occurring in open channel flows.

The flow in a pressure conduit is confined by solid walls on every side, while the flow in an open channel has a free surface on one side. When the flow in a pressure conduit is strongly disturbed by high speed or a high ratio of relative roughness, the disturbance is extended over the entire domain in the cross section. In this case, the zero-velocity point at which the logarithmic velocity vanishes determines the logarithmic velocity profile (Yoo, 1993a). It becomes almost constant and is mainly associated with the wall roughness. The flow then reaches a stage of RT flow; that is, it is restricted from further stretching. However, when the free surface flow is disturbed for any reason and the disturbance is extended over a cross section, the surface may rise and fall because it is free to move.

When pipes are not filled, the flows are considered open channel flow. Sewers and drainage culverts may come under this classification. The factors affecting losses of fluid movement in conduits are almost independent of pressure, and hence, the same laws may apply to the flow of water in both pipes and open channels. However, there are some differences between them, because an interaction among solid, air, and liquid (SAL contact) is added to the open channel flow. The contact area between air and liquid (AL) is considered independent of frictional losses, and the surface is normally denoted a free surface. However, both SAL contact and AL contact on the free surface may play roles on the transport of water in open channels.

Although the existence of the free surface may cause the open channel flow to be different from the pressure conduit flow in many aspects, the distinction between these flows has not been clearly made by sufficiently noting the differences in their flow characteristics. For example, several empirical equations of mean velocity are mutually employed for the description of both pipe flow and open channel flow, even though the computations sometimes result in different values.

Empirical equations of mean velocity or friction factor have been suggested by various researchers, including Bazin (1865), Ganguillet and Kutter (1869), Manning (1891), and Williams

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and Hazen (1933). The Hazen-Williams equation is most commonly employed for flow in pressure conduits, particularly for commercial (connected) pipes, while the others are primarily used for flow in open channels. However, the Manning equation has been suggested for pipe flow, and the Hazen-Williams equation has been suggested for open channel flow. The major factor for this multiuse is that both equations use the data for pipe flow and open channel flow, and no particular difference can be found between them in their analyses. In the present study, the existing equations are reviewed to determine whether any trend distinguishes the special features of open channel flow in comparison with pipe flow. Furthermore, several sets of laboratory data are reanalyzed to make the difference between pipe flows and open channel flows clear.

For open channel flow, the effect of secondary loss is often considered to be included in the effect of the friction factor, and $h_L = h_{fL}$. Therefore:

$$h_L = h_{fL} = \Delta z = f\frac{L}{d}\frac{V^2}{2g} = \frac{f}{8}\frac{L}{R}\frac{V^2}{g} = C_f\frac{L}{R}\frac{V^2}{g} \tag{1}$$

and

$$V = \sqrt{\frac{8g}{f}}\sqrt{Rs} = \sqrt{\frac{g}{C_f}}\sqrt{Rs} = C_h\sqrt{Rs} \tag{2}$$

where, h_L is the head loss, h_{fL} is the frictional head loss, f is the Darcy-Weisbach friction factor, $C_f(=f/8)$ is the resistance friction factor, L is the channel length, d is the diameter of circular pipe, g is the acceleration because of gravity, V is the cross-sectional mean velocity, s is the energy slope defined by $s = \Delta z/L$ for steady and uniform flow, Δz is the elevation difference between two points, and $C_h\left(=\sqrt{\frac{g}{C_f}}\right)$ is the Chezy friction factor. For a uniform steady flow, the friction factor C_f has no dimension, while C_h has the dimension $[L^{0.5}T^{-1}]$ and R is the hydraulic radius defined as:

$$R = \frac{A}{P} \tag{3}$$

where, P is the wetted perimeter and A is the cross-sectional area. For circular pipe flow, $R = r/2 = d/4$ where, r and d are the radius and diameter of the circular pipe, respectively. The friction factor $C_p(=C_f)$ of pipe flow is defined by:

$$\tau = \rho C_f V^2 \quad \text{or} \quad C_p = \frac{\tau}{\rho V^2} \tag{4}$$

where, τ is the shear stress at the wall.

Equation (2) is called the Chezy equation. One of the major factors affecting open channel flows is the relative roughness or the ratio of pipe diameter to the roughness as used in the description of pipe flow, but other factors are not yet clearly defined.

METHODS

In this section, the fundamentals of the friction factor in circular pipe flow and empirical equations of mean velocity of uniform channel flow are presented. The new open channel friction factors of unsteady, nonuniform and steady, uniform flow are developed using the logarithmic law based on pipe flow.

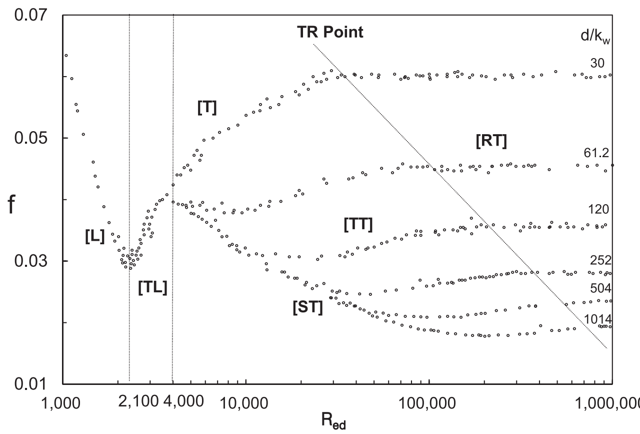


Figure 1. Distribution of the Darcy-Weisbach friction factor $f(=8C_f)$ against the pipe diameter Reynolds number R_{ed} with laboratory data from Nikuradse (1933). The friction factor in ST flow is stretched with Reynolds number. However, in the RT flow region, the friction factor is restricted and is proportional to the roughness height k_w. The relative roughness, or the ratio of pipe diameter to the roughness, d_k is one of the most important friction factors in the RT flow region for both pipe flow and open channel flow (Yoo, 1993b).

Friction Factor in Circular Pipe Flow

The flow characteristics of an open channel are considered different from those of a circular pipe; however, it is considered that they share some common aspects of flow mechanism, and basically, open channel flow is similar to pipe flow. For circular pipe flow, the logarithmic friction factor (Yoo and Lee, 1999) is described by integrating the velocity over the cross section and is given by:

$$\frac{1}{\sqrt{C_p}} = \frac{1}{k}\left(\ln\frac{r}{z_o} - 1.5\right) \tag{5}$$

or

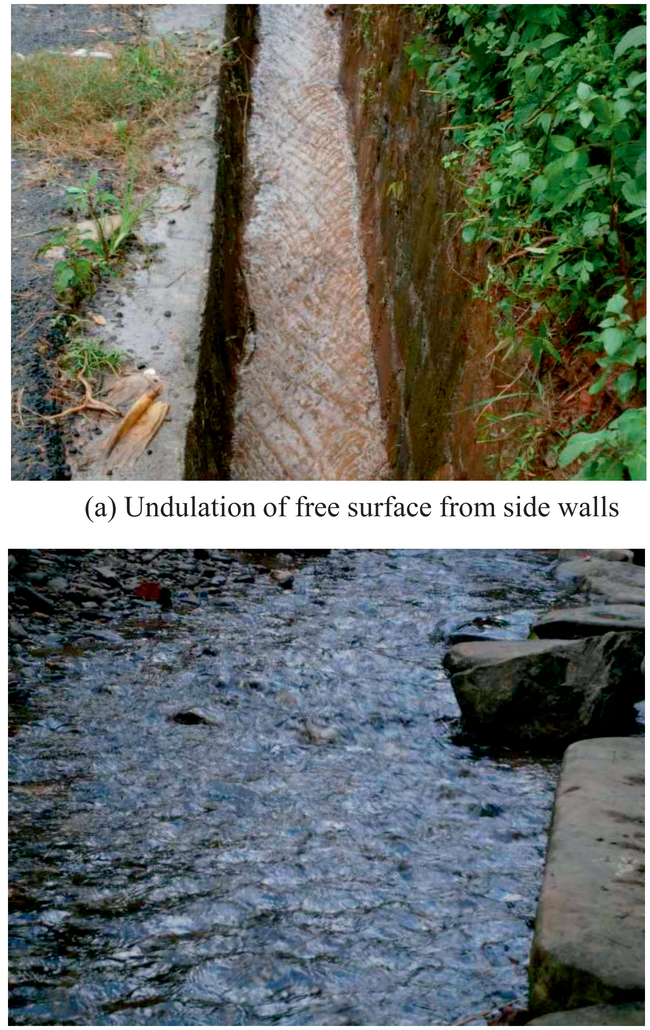
$$\frac{1}{\sqrt{C_p}} = \frac{1}{k}\left(\ln\frac{R}{z_o} - 0.8\right) \tag{6}$$

where, z_o is the zero-velocity point at which the logarithmic velocity vanishes, $\sqrt{C_p} = u_*/V$ (u_* is the shear velocity defined by $\sqrt{\tau/\rho}$ where, ρ is the water density).

Nikuradse (1933) conducted experiments with the roughness made by attaching uniform sands inside a pipe. He devised six sets of the roughness ratio $d_k(=d/k_w)$: 30, 61.2, 120, 252, 504, and 1014, as shown in Figure 1. The roughness height k_w is defined as the mean diameter of sands attached inside the pipe, which is called the Nikuradse equivalent roughness height. Even when the roughness heights are different, but the roughness ratio is the same, the distribution of the friction factor is located on the same line. It is therefore concluded that the roughness ratio is one of the most important factors affecting the frictional characteristics of pipe flow. However, the roughness height k_w might be important, as well as the roughness ratio d_k, for open channel flow.

Turbulent flow occurs when the pipe diameter Reynolds number $R_{\rm ed}$ exceeds 4000. Then, the friction factor in smooth turbulent (ST) region decreases with the Reynolds number up to a certain point at which the decrease rate reduces, and it then increases in the transitional turbulent (TT) region with the Reynolds number. However, the increase rate gradually reduces in the rough turbulent (RT) region as the Reynolds number increases, and finally, the friction factor becomes constant, irrespective of the Reynolds number. The first part of turbulent flow is called the ST flow region, the second part is the TT flow region, and the final part is the RT flow region. It is generally considered that the flow in the ST region is smoother than that in the RT region. However, there is no measure of the flow roughness height k_w, and it is expected that the zero-velocity point z_o in RT would also be constant and steady, similar to that of the zero-velocity point in ST flow (Yoo and Lee, 1999). The zero-velocity point in RT flow is approximately 10 μm, which is smaller than the height of undulation found at the free surface, and hence, the condition of RT flow is as smooth as that of ST flow. The height of free surface undulation is approximately 1 cm, as shown in Figure 2. Therefore, the roughness of free surface undulation is almost 1000 times greater than that of the zero-velocity point in RT flow. The significant difference between ST and RT is only that the zero-velocity point is stretching in the ST region, while the zero-velocity point becomes restricted and proportional to the roughness height in the RT region. Therefore, it is better to define ST as "stretching turbulent" and RT as "restricted turbulent." In the ST region, the velocity profile continuously stretches, and hence the zero-velocity point continues to decrease with the Reynolds number or velocity. However, in the RT region, the stretching of the velocity profile is restricted by the limited space, and hence the zero-velocity point becomes constant and proportional to the roughness height. To stretch the zero-velocity point, plenty of space should be allowed so that the velocity profile could be elongated freely. To restrict the zero-velocity point, space for the development of flow should be limited so that d_k is small.

The point between the laminar (L) flow region and transitional laminar (TL) flow region is called the LT point; the TS point is between TL and ST, the ST point is between ST and TT, and the TR point is between TT and RT. The boundary points can be determined by the pipe diameter Reynolds number $R_{\rm ed}$, and as expected, $R_{\rm ed-LT} = 2000$ and $R_{\rm ed-TS} = 4000$. It is widely suggested that $R_{\rm ed-ST}$ and $R_{\rm ed-TR}$ may be determined by introducing the shear velocity Reynolds number $R_{e*}(=u_* k_w/\upsilon)$ where, u_* is the shear velocity, k_w is the roughness height, and υ is the kinematic viscosity of water. However, many researchers suggested different values for $R_{e*-\rm ST}$, from 3 to 7, and for $R_{e*-\rm TR}$, from 35 to 110. The values are found to vary with different roughness ratio $d_k(=d/k_w)$ or $R_k(=R/k_w)$. Instead of R_{e*}, the pipe diameter Reynolds number R_e at the boundary point is found to be proportional to the relative roughness ratio (Yoo, 1993a, 1993b). Each region of the flow condition for circular pipes is suggested as follows:



(a) Undulation of free surface from side walls

(b) Undulation of free surface from bottom

Figure 2. Undulation effect on open channel flow: (a) undulation of the free surface from the side walls and (b) undulation of the free surface from the bottom.

L: $R_{\rm ed} < 2000$ ($R_{\rm eR} < 500$)
TL: $2000 < R_{\rm ed} < 4000$ ($500 < R_{\rm eR} < 1000$)
ST: $4000 < R_{\rm ed} < 81.5d_k$ ($1000 < R_{\rm eR} < 81.5R_k$)
TT flow: $81.5d_k < R_{\rm ed} < 1140d_k$ ($81.5R_k < R_{\rm eR} < 1140R_k$)
RT: $1140d_k < R_{\rm ed}$ ($1140R_k < R_{\rm eR}$)

where, the pipe diameter Reynolds number is $R_{\rm ed} = Vd/\upsilon$, the hydraulic radius Reynolds number is $R_{\rm ed} = VR/\upsilon$, and $R_{\rm eR} = R_{\rm ed}/4$. The critical values of $R_{\rm ed-ST}$ and $R_{\rm ed-TR}$ are found to be proportional to the relative roughness $d_k(=d/k_w)$. The proportional factors, 81.5 and 1140, are valid only for flow in circular pipes. They could vary for different shapes of pipe. Several facts should be noted from the Nikuradse laboratory experiments. If the roughness ratio $\epsilon\left(=\frac{k_w}{d}=d_k^{-1}\right)$ is large or d_k is small, the flow can be RT with a small Reynolds number. However, even if the ratio ϵ is very large, the flow will still be L when the Reynolds number is less than 2000.

By employing the laboratory data obtained by Nikuradse (1933) for the ST flow, Nikuradse suggested that:

$$\frac{1}{\sqrt{f}} = 2\log R_{\mathrm{ed}}\sqrt{f} - 0.8 = 2\log R_{\mathrm{ed}}^{0.89} - 1.42 \tag{7}$$

where, $f = 8C_p$. The last expression of Equation (7) approximates the first expression. Using the friction factor C_p of pipe flow (Yoo and Lee, 1999):

$$\frac{1}{\sqrt{C_p}} = \frac{1}{k}(\ln R_{\mathrm{eR}}\sqrt{C_p} + 1.5) \tag{8}$$

Substituting Equation (5) into Equation (7) yields the zero-velocity point for the ST flow as follows:

$$z_o = 0.1\nu/u_*$$

The approximate explicit equation of the friction factor C_p is given from the last expression of Equation (7) as follows:

$$\frac{1}{\sqrt{C_p}} = \frac{1}{k}(\ln R_{\mathrm{eR}}^{0.89} - 0.39) \tag{9}$$

Empirical Equations of Mean Velocity for Uniform Channel Flow

Based on the Chezy equation in Equation (2), several researchers developed empirical equations, by primarily adjusting the values of exponents to R and s or representing the Chezy friction factor C_h with various forms. As an initial approach, several researchers proposed determining the exponent by regression analysis, and the Manning-Strickler (MS) equation is represented by (Strickler, 1923):

$$V = \frac{1}{n}R^{0.67}s^{0.5} \tag{10}$$

Bazin (1865) suggested that the friction factor of an open channel is primarily related to the relative roughness ratio and argued that the channel slope has a minor effect on the flow. He developed his empirical equation using his own data, of which the Reynolds number ranges from 40,000 to 600,000. Later, the empirical coefficient in Equation (10) was determined using Bazin's data. Therefore, the MS equation is considered valid primarily for the second group of stretching (smooth) turbulent flow (ST II). However, Ganguillet and Kutter (1869) have determined the parameters using the discharge data from the Mississippi River and from various natural and artificial channels in Europe. The Ganguillet and Kutter (GK) equation is represented by:

$$C_h = \alpha_{\mathrm{GK}}\frac{1}{n} \tag{11}$$

where,

$$\alpha_{\mathrm{GK}} = \frac{1 + \xi nX}{1 + nX/\sqrt{R}} \quad \text{with} \quad X = 23 + \frac{0.0015}{s}$$

where, n is Kutter's roughness factor and ξ is an adjusting factor for the size of channel. It is recommended that $\xi = 1.0$ for small streams and $\xi = 0.55$ for large rivers. However, the variation of ξ with the size of channel is not clearly defined. The GK equation shows that the Chezy factor is a function of the channel slope X, roughness ratio $n/\sqrt{R}$, and roughness n, while the MS equation shows that the Chezy friction factor C_h is only a function of the relative roughness or roughness ratio ϵ. The Chezy factor is simply replaced by $1/n$, assuming the variation of α_{GK} could be compensated for by the power of the hydraulic radius or other uncertainties.

The friction factor in Equation (1) is associated with the MS equation in Equation (10), and the friction factor is found to be related only to the roughness ratio ϵ as follows:

$$C_f = \frac{gn^2}{R^{0.33}} = \xi\epsilon^{-0.33} \tag{12}$$

where, $\epsilon = k_w/R$, assuming $9.8n^2 = \xi k_w^{0.33}$ in International System (SI) units. The proportionate factor in Equation (12) was normally estimated between 0.016 and 0.022, depending on the physical condition of open channel flow, but its optimum value is considered to vary with the channel size and flow condition.

Friction Factor of Restricted Open Channel Flows

Bazin (1865) took measurements of open channel flow in a wide and long artificial channel with a width of 2 m and length of 600 m. The water depth ranged up to approximately 0.25 m. In most of his experiments, the shape factor $s_h(=h/b)$ ranged from 0.02 to 0.1. The roughness of the wall was made artificially using natural materials, such as mortar, concrete, wood plates, and bricks. The Nikuradse equivalent roughness heights were not yet defined. Bazin tested the experiments on channels of five different slopes, but so far, the effects of channel slope have not been considered. The channel slope was fixed, and only the discharge was varied. Because his channel was long enough, at more than 600 m, uniform flow was developed in sufficient length. In contrast, Varwick (1945) conducted elaborate laboratory experiments on two small-scale channels of different shapes: one was trapezoidal, and the other was triangular. Both slope and discharge were elaborately adjusted so that constant values of the roughness ratio were obtained, as done by Nikuradse (1993) in his experiments on pipe flow. Both experimental results were plotted on a logarithmic scale by Kirschmer (1949), as shown in Figure 3. The laboratory data on L flow conducted by Straub, Silberman, and Nelson (1958) are included in the same graph. The graph shows the distribution of the friction factor against the hydraulic radius Reynolds number.

The laboratory results of Varwick (1945) show a clear trend of the ST flow condition in most cases, and those of Bazin (1865) show the definite characteristics of the ST flow condition. They are clearly shown on the graph of the friction factor against the Reynolds number presented by Kirschmer (1949). Before Kirschmer summarized the laboratory results of Varwick and Bazin, Keulegan (1938) estimated the roughness height of each channel, based on the assumption that the flow in Bazin's channel is RT. However, the assumption of RT flow in Bazin's channel violates Varwick's definition of channel flow. Similar to the friction factor of pipe flow, the friction factor of open channel flow consists of L, ST, TT, and RT regions. The boundary ST or TR points of the Reynolds number move

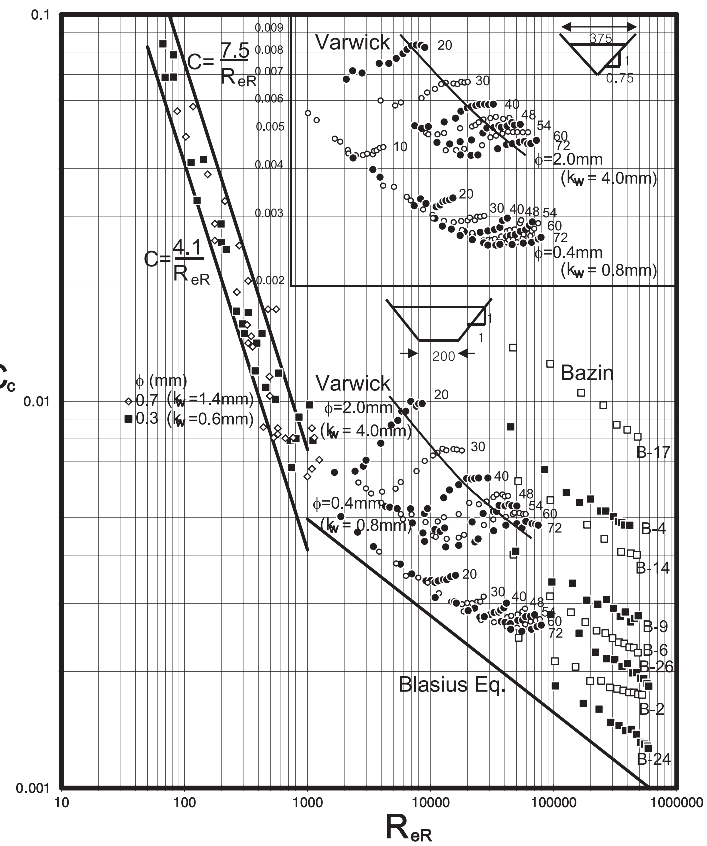


Figure 3. Friction factor C_c against hydraulic radius Reynolds number R_{eR}, with laboratory experiment data from Bazin (1865), Varwick (1945), and Straub, Silberman, and Nelson (1958).

slightly, with the roughness height, from the values of the pipe friction factor.

The results of the three laboratory experiments conducted by Bazin (1865), Varwick (1945), and Straub, Silberman, and Nelson (1958) demonstrate that the open channel flow can be L when the Reynolds number is less than 1000, and in most cases, the flow conditions are found to be ST. The friction factor of open channel flow decreases as the Reynolds number increases, but with a different magnitude of the proportionating factor. RT flow may occur, but only when the flow is strongly restricted by the channel boundary. As clearly observed from the laboratory data of Varwick (1945), the friction factor of open channel flow also decreases as the Reynolds number increases at the condition of ST flow; however, it increases with the roughness k_w, not with the roughness ratio. This is a dominant aspect of open channel flow, which is strikingly different from that of pipe flow. The friction factor of pipe flow is determined by the relative roughness. Even if the pipe internal wall has a different roughness but the same roughness ratio, the distribution of the pipe flow friction factor is found on the same line. However, if the open channel roughness conditions are different with the same roughness ratio, the distribution of the open channel flow friction factor is found on different lines. The friction factor of higher roughness is located above that of lower roughness, even if both have the same roughness ratio. This may be largely because of the surface undulation caused by roughness, as shown in Figure 3. Furthermore, the existing equations, such as the MS equation, the GK equation, and Bazin's equation, cannot properly describe the laboratory data shown in Figure 3.

As shown in Figure 3, the flow condition of an open channel is different from that of pipe flow, and each region of flow condition for an open channel is suggested as follows:

L: $R_{\mathrm{eR}} < 1000$
ST: $1000 < R_{\mathrm{eR}} < R_{\mathrm{eR\text{–}ST}}$
TT flow: $R_{\mathrm{eR\text{–}ST}} < R_{\mathrm{eR}} < R_{\mathrm{eR\text{–}TR}}$
RT: $R_{\mathrm{eR\text{–}TR}} < R_{\mathrm{eR}}$
Triangular channel: $R_{\mathrm{eR\text{–}ST}} = (-15 + 37k_w)R_k^{0.75}$, $R_{\mathrm{eR\text{–}TR}} = 130R_k^{0.75}$
Trapezoidal channel: $R_{\mathrm{eR\text{–}ST}} = (10 + 8.5k_w)R_k^{0.75}$, $R_{\mathrm{eR\text{–}TR}} = 130R_k^{0.75}$

The effect of parallel increase has been represented by changing the slope and the intercept (Yoo and Lee, 1999). For ST flow, the friction factor C_c of open channel flow is given by:

$$\frac{1}{\sqrt{C_c}} = \frac{\alpha}{k}(\ln R_{\mathrm{eR}}\sqrt{C_c} + \beta) \tag{13}$$

Table 1. *Manning coefficient* n *and increment factor* ζ *for various channels with different surface materials and states*.

Channel Type	Surface Materials	State	n	ζ
Models	Mortar	Straight	0.011	1.2
	Perspex	Straight	0.009	1.0
Lined canals	Concrete	Straight	0.012	1.5
Unlined canals	Earth	Straight	0.018	3
	Rock	Straight	0.025	6
Rivers	Earth	Straight	0.020	4
		Meandering	0.030	9
	Gravel	Straight	0.030	9
		Winding	0.040	16–64
	Grass	Straight	0.060	36–144

For circular pipe flow, α = 1 and β = 1.5, as given by Equation (8). However, for open channel flow, it is found that both α and β change with the channel shape and roughness. Yoo and Lee (1999) proposed relating α to the channel shape and β to the roughness height, using the data shown in Figure 3. Partial success has been achieved by employing Equation (13). However, general relations have not been made for the expression of slope α and intercept β. Furthermore, it was not possible for the logarithmic equation to be associated with the traditional roughness factor n. This is because Equation (13) is based on ST flow, while the MS equation is based on RT flow. Thus, it was not possible for the roughness height to be related to the factor n. The present study proposes to relate the roughness to the Manning coefficient.

The frictional motion of open channel turbulent flow is complicated, and because of this complication, the resulting friction factor gradually varies with any condition. As the Reynolds number increases, the friction factor of open channel flow also decreases, similar to that of pipe flow. This condition may be called ST flow. The friction factor of open channel flow at a certain condition can also be categorized as TT flow or RT flow, as found from Varwick's laboratory data. However, the strikingly different feature of the open channel flow friction factor is that it moves upward from the distribution of the pipe flow friction factor. This may be largely because of the shape distortion of the velocity profile and the undulation of the free surface. That is, the friction factor increases, but the general trend of its distribution is logarithmic, as given by Prandtl's (1925) mixing length theory. It is now suggested as:

$$C_c = \zeta C_p \tag{14}$$

where, the subscript c indicates open channel flow and ζ is the increment factor.

From Equations (14) and (12), the relation between n and α_c or ζ is given by:

$$\zeta = \frac{g}{R^{0.33} C_p} n^2 \tag{15}$$

The magnitude of the Kutter or Manning coefficient has been determined using Bazin's data, which are mostly in ST II, considering the values of C_p and R for the ST II region. From Equations (10) and (14) and considering $C_p = 0.012 R_{\mathrm{eR}}^{-0.17}$ for the ST II region, the relation between n and α_c or ζ is given by:

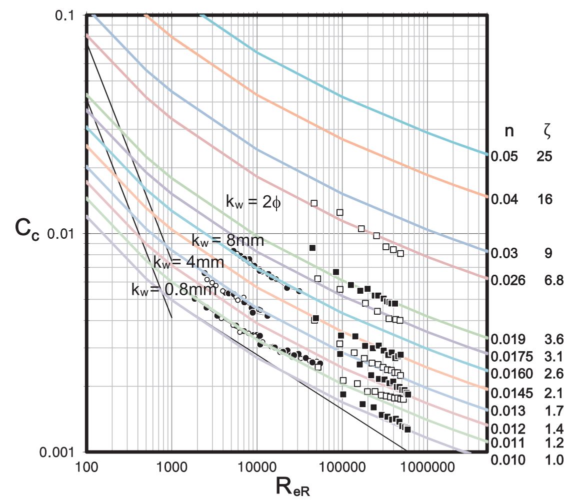


Figure 4. Friction factor C_c in Equation (18) against hydraulic radius Reynolds number R_{eR} for unsteady, nonuniform, open channel RT flows.

$$\zeta = \frac{g R_{\mathrm{eR}}^{0.17}}{0.012 R^{0.33}} n^2 \tag{16}$$

The Reynolds number R_{eR} ranges from 40,000 to 2×10^6 for the ST II region. Assuming $R = 0.1$ m at $R_{\mathrm{eR}} = 40{,}000$, $\zeta = 10{,}590 n^2$. For $R = 0.3$ m at $R_{\mathrm{eR}} = 400{,}000$, $\zeta = 10{,}900 n^2$, and for $R = 0.5$ m at $R_{\mathrm{eR}} = 1{,}000{,}000$, $\zeta = 10{,}750 n^2$. Thus, ζ is can be reasonably estimated as:

$$\zeta \cong 10{,}000 n^2 \tag{17}$$

The values of the increment factor ζ for various channels are presented in Table 1.

Assuming $C_c = \zeta C_p$ and $\zeta = 10^4 n^2$, the new equation for the logarithmic friction factor, the Yoo and Lim (YL) equation, in unsteady, nonuniform, open channel RT flow is obtained as:

$$\frac{1}{\sqrt{C_c}} = \frac{0.01}{n}(2.2 \ln R_{\mathrm{eR}} - 1) \tag{18}$$

A comparison between laboratory data and computational results from Equation (18) is shown in Figure 4. To clarify the comparison, the laboratory data of L flow and TL flow are excluded. The equation of unsteady flow in Equation (18) is employed for the computation, because the major parameter is RT and no slope is denoted.

Friction Factor of Stretching Open Channel Flows

For uniform and steady flow, Equation (8) is further extended to compute the mean velocity. The base friction factor is the circular pipe friction factor C_p, and the logarithmic law is given by Equation (8). ST flow is represented by Equation (8). For uniform steady flow, the friction factor is derived from Equation (2) as:

$$R_{\mathrm{eR}}\sqrt{C_p} = \frac{VR}{v}\sqrt{C_p} = \sqrt{\frac{gRs}{C_p}}\frac{R}{v}\sqrt{C_p} = \frac{R\sqrt{gRs}}{v} \tag{19}$$

where, $V = \sqrt{gRs/C_p}$ for uniform and steady flow. Substituting Equation (19) into the right-hand side of Equation (8) yields:

$$\frac{1}{\sqrt{C_p}} = \frac{1}{k}\left(\ln\frac{R\sqrt{gRs}}{\nu} + 1.5\right)$$

The friction factor C_p is given by Equation (14), that is, $C_p = C_c/\zeta$. Then, the new equation for the logarithmic friction factor in steady, uniform open channel flow is obtained as:

$$\frac{1}{\sqrt{C_c}} = \frac{1}{k\sqrt{\zeta}}\left(\ln\frac{R\sqrt{gRs}}{\nu} + 1.5\right) \quad (20)$$

Assuming $\zeta = 10{,}000n^2$, as suggested in Equation (16); $g = 9.8$ m/s^2; and $\nu = 10^{-6}$ m^2/s, the new logarithmic equation (YL) of mean velocity for open channel ST flow is finally obtained as:

$$V = \frac{0.078\sqrt{Rs}}{n}\left[\ln R\sqrt{Rs} + 16.4\right] \quad (21)$$

Equation (21) should be used only for steady, uniform open channel flow, while Equation (18) can be used for unsteady, nonuniform, open channel flow.

RESULTS

The new logarithmic equation in Equation (21) of the mean velocity in open channel flow is valid for a range of Reynolds numbers from 1000 to possibly 1 trillion, which may occur in wide rivers, as long as the flows somehow have logarithmic profiles. Open channel flows are considered to be primarily ST and have the logarithmic function of the Reynolds number. However, because of the existence of the free surface, a large increment is considered to be imposed on the energy dissipation or frictional head loss for the case of open channel flow. The mean velocity of the new logarithmic equation in Equation (21) in open channel flow is compared to the MS equation in Equation (10) and GK equation in Equation (11) for three cases of rectangular open channel flow in the following example.

Example

Estimate the cross-sectional mean velocity of rectangular open channel flow as follows:

(1) $h = 0.1$ m, $b = 2$ m, $s = 0.0009$, $n = 0.02$
(2) $h = 2$ m, $b = 4$ m, $s = 0.0009$, $n = 0.02$
(3) $h = 20$ m, $b = 300$ m, $s = 0.0004$, $n = 0.04$

Case 1

$$R = \frac{bh}{b+2h} = \frac{0.1\times 2}{2+0.2} = 0.091 \text{ m}$$

By logarithmic equation (YL):

$$\sqrt{Rs} = \sqrt{0.091\times 0.0009} = 0.00909$$

$$V = \frac{0.078\times 0.009}{0.02}[\ln(0.091\times 0.009) + 16.4] = 0.33 \text{ m/s}$$

By the MS equation:

$$V = \frac{\sqrt{s}}{n}R^{0.67} = \frac{\sqrt{0.0009}}{0.02}\times 0.091^{0.67} = 0.30 \text{ m/s}$$

By the GK equation:

$$X = 23 + \frac{0.0015}{s} = 23 + \frac{0.0015}{0.0009} = 24.67$$

$$\alpha_{\mathrm{GK}} = \frac{1+\xi nX}{1+nX/\sqrt{R}} = \frac{1+1\times 0.02\times 24.7}{1+0.02\times 24.7/\sqrt{0.091}} = \frac{1.49}{2.638} = 0.565$$

$$V = \frac{\alpha_{\mathrm{GK}}}{n}\sqrt{Rs} = \frac{0.565}{0.02}\sqrt{0.091\times 0.0009} = 0.26 \text{ m/s}$$

For laboratory channel flows, the MS equation provides a value of 0.3 m/s for the mean velocity, which is approximately 7.5% lower than that of logarithmic law; however, the GK equation provides a large underestimation of approximately 21.9% in comparison with that of the logarithmic law.

Case 2

$$R = \frac{bh}{b+2h} = \frac{2\times 4}{4+4} = 1.0 \text{ m}$$

By the YL equation:

$$\sqrt{Rs} = \sqrt{1\times 0.0009} = 0.03$$

$$V = \frac{0.078\times 0.03}{0.02}[\ln(1\times 0.03) + 16.4] = 1.51 \text{ m/s}$$

By the MS equation:

$$V = \frac{\sqrt{s}}{n}R^{0.67} = \frac{\sqrt{0.0009}}{0.02}\times 1.0^{0.67} = 1.50 \text{ m/s}$$

By the GK equation:

$$X = 23 + \frac{0.0015}{s} = 23 + \frac{0.0015}{0.0009} = 24.67$$

$$\alpha_{\mathrm{GK}} = \frac{1+\xi nX}{1+nX/\sqrt{R}} = \frac{1+1\times 0.02\times 24.67}{1+0.02\times 24.67/\sqrt{1.0}} = \frac{1.49}{1.49} = 1.0$$

$$V = \frac{\alpha_{\mathrm{GK}}}{n}\sqrt{Rs} = \frac{1.0}{0.02}\sqrt{1\times 0.0009} = 1.50 \text{ m/s}$$

Both the MS equation and the GK equation provide similar values of mean velocity, which have an underestimation of about 0.6% in comparison with that of the logarithmic law for small channels.

Case 3

$$R = \frac{bh}{b+2h} = \frac{20\times 300}{300+40} = 17.65 \text{ m}$$

By YL equation:

$$\sqrt{Rs} = \sqrt{17.6\times 0.0004} = 0.084$$

$$V = \frac{0.078\times 0.084}{0.04}[\ln(17.6\times 0.084) + 16.4] = 2.75 \text{ m/s}$$

By MS equation:

$$V = \frac{\sqrt{s}}{n}R^{0.67} = \frac{\sqrt{0.0004}}{0.04}\times 17.6^{0.67} = 3.39 \text{ m/s}$$

Table 2. *Comparison of the two empirical equations, the MS equation in Equation (10) and the GK equation in Equation (11), with the new logarithmic equation in Equation (21) for several cases of steady, uniform channel ST flow condition.*

							V (m/s)			
Case	h (m)	b (m)	s	n	R	R_{eR}	YL	MS	GK	ξ
1	0.05	1.0	0.0009	0.02	0.045	9.4e3	0.21	0.19	0.17	1.5
2	0.10	2.0	0.0009	0.02	0.091	3.0e4	0.33	0.30	0.30	1.5
3	1.0	3.0	0.0009	0.02	0.60	6.6e5	1.10	1.07	1.06	1
4	2.0	4.0	0.0009	0.02	1.00	1.5e6	1.51	1.50	1.50	1
5	3.0	6.0	0.0009	0.02	1.50	2.9e6	1.93	1.97	1.96	1
6	5.0	100.0	0.0004	0.04	4.55	5.6e6	1.23	1.37	1.13	0.55
7	10.0	200.0	0.0004	0.04	9.10	1.7e7	1.86	2.18	1.77	0.55
8	20.0	300.0	0.0004	0.04	17.65	4.9e7	2.75	3.39	2.66	0.55
9	30.0	500.0	0.0004	0.04	26.79	9.4e7	3.52	4.48	3.41	0.55

By GK equation:

$$X = 23 + \frac{0.0015}{s} = 23 + \frac{0.0015}{0.0004} = 26.75$$

$$\alpha_{GK} = \frac{1 + \xi n X}{1 + nX/\sqrt{R}} = \frac{1 + 0.55 \times 0.04 \times 26.75}{1 + 0.04 \times 26.75/\sqrt{17.6}} = \frac{1.59}{1.255} = 1.267$$

$$V = \frac{\alpha_{GK}}{n}\sqrt{Rs} = \frac{1.267}{0.04}\sqrt{17.6 \times 0.0004} = 2.66 \text{ m/s}$$

For wide rivers, the GK equation provides similar values to those of the new logarithmic law if the factor ξ is reduced to 0.55 from 1.0. In comparison with the new YL equation, the GK equation has approximately 3.3% underestimation, while the MS equation produces a large overestimation of approximately 23.2%. This is a significant underestimation of water depth for a given discharge, which may cause serious floods over dikes.

Test of Example

The three cases exemplified in the preceding example are tested using the two existing empirical equations in comparison with the new logarithmic law. The results are presented in Table 2. As shown in the table, the GK equation provides similar values of mean velocity in comparison with those of the YL equation for relatively small rivers with a value of $\xi = 1.0$ and for wide rivers with a value of $\xi = 0.55$. The reason for the reduction in the factor ξ is considered mainly because of further stretching of the ST logarithmic profile in large rivers, as Ganguillet and Kutter (1869) experienced in some tests in the Mississippi River. The factor ξ could be further reduced for extremely long and wide rivers, such as the Nile River or the Mekong River. The MS equation provides similar values of mean velocity to those of the YL equation for relatively small rivers; however, it provides larger values than those of the YL equation or GK equation for big rivers, because the equation or the Manning factor is primarily determined using data from small channels. The MS equation provides a large overestimation of the mean velocity in comparison with the YL equation for large rivers or for hydraulic conditions with Reynolds numbers larger than 10^7.

As presented in Table 2, the present YL equation in Equation (21) for ST flow provides similar results to the MS equation in Equation (10) and the GK equation in Equation (11) for steady and uniform flows. For small streams, where the hydraulic radius Reynolds number is expected to be small, the YL equation provides values of mean velocity similar to those of the MS equation, while the GK equation provides smaller values of mean velocity compared with those of the YL equation. However, for large rivers, where the Reynolds number is expected to be very large, the YL equation provides values of mean velocity similar to those of the GK equation, while the MS equation provides larger values of mean velocity compared with those of the YL equation and GK equation. The MS equation was initially developed using Bazin's data (1865), which is categorized as ST II. The GK equation was initially developed using the data from small streams, and the factor ξ of the equation has been adjusted from 1 for small streams and to 0.55 for large rivers. Therefore, the present logarithmic equation based on ST flow is generally coincident with existing experiences.

Figure 5 shows a comparison of computational results using the MS, GK, and YL equations for various conditions of $n = 0.02$ and $n = 0.04$ and of $s = 0.0004$ and $s = 0.0009$. The hydraulic radius R is allowed to vary from 0.5 to 30 m, which is considered to cover the range of the first to fourth groups of stretching turbulent flow (ST I to ST IV). As shown in the graph, the MS equation yields small computational results of mean velocity similar to those of the YL equation when the flow condition is restricted to ST I. However, the MS equation yields very large computational results of mean velocity in comparison with those of the YL equation when the flow condition reaches ST IV. The empirical factor ξ of the GK equation in Figure 5 uses 1.0 from ST I to ST IV. For wide rivers such as the Mississippi river, where n is large, the GK equation also overestimates the mean velocity.

DISCUSSION

It is proposed that the ST flow should be short for "stretching turbulent" flow instead of "smooth turbulent" flow and RT flow should be short for "restricted turbulent" flow instead of "rough turbulent" flow. Both are smooth for the case of pressurized pipe flow compared with the undulating motion of the free surface in open channel flow, which may be called rough- or undulating-surface ST flow. Although the undulation of the free surface is one of the factors for energy dissipation or head loss, the main body in open channel flow acts as pressurized pipe flow. Therefore, the friction factor decreases logarithmically (as in pipe flow), which could be represented by Prandtl's law. This is valid for external flow, as well as for confined pressurized pipe flow.

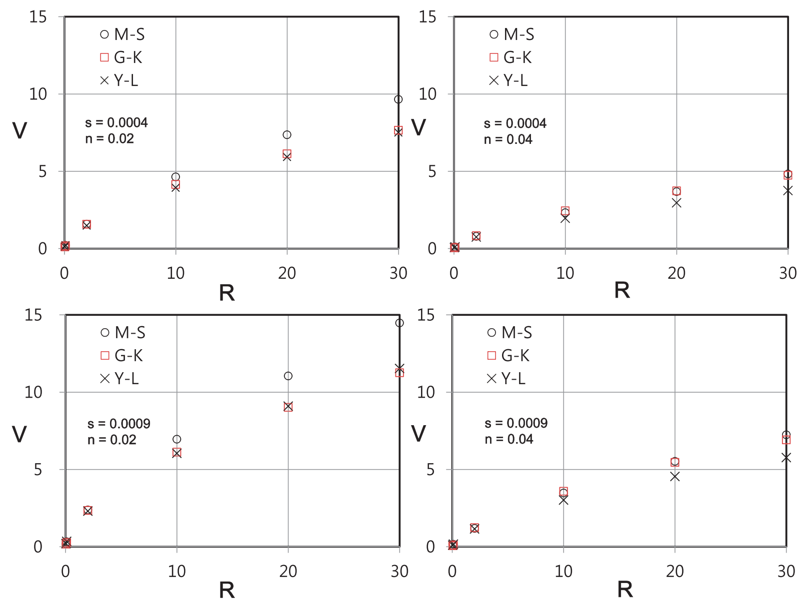


Figure 5. Comparison of computed mean velocity V against hydraulic radius R with different n and s values using the MS equation in Equation (10), GK equation in Equation (11), and YL equation in Equation (21) for steady, uniform ST open channel flows.

CONCLUSION

The logarithmic laws could be adopted in a range of open channel turbulent flows. The new friction factor for undulating-surface (rough surface) ST flow is found to be similar to the results of the MS equation for small streams and to those of the GK equation for wide rivers. The inclusion of the slope effect is considered possible only when the friction factor is strongly associated with the hydraulic radius Reynolds number. This means that in most cases, open channel flows are ST. RT may hardly occur in open channel flows, because the logarithmic profile would not be restricted sufficiently when the flow has a free surface.

The GK equation and the new YL equation may have to be adopted for open channel flow for big rivers. However, the validity of the GK equation is doubtful for extremely wide rivers that may experience very large Reynolds numbers.

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LITERATURE CITED

Bazin, H.E., 1865. Recherches experimentales sur lecoulement de leau dans les canaux decouverts. *Memories presents par divers savants al Academie des Sciences* (Paris), p. 19 (in French).

Ganguillet, E. and Kutter, W.R., 1869. An investigation to establish a new general formula for uniform flow of water in canals and river. *Zeitschrift des Oesterriechischen Ingenieur- und Architekten-Vereines,* 21(2), 129–147.

Keulegan, G.H., 1938. Laws of turbulent flow in open channels. *Journal of Research of the National Bureau of Standards,* 21, 707–741.

Kirschmer, O., 1949. Reibungsverluste in Rohren und Kanalen. *Die Wasserwirtschaft,* 39(7), 8 (in German).

Manning, R., 1891. On the flow of water in open channels and pipes. *Transactions of the Institution of Civil Engineers of Ireland,* 20, 161–207.

Nikuradse, J., 1933. Stroemungsgesetze in rauhen Rohren. *VDI-Forschungsheft,* (Berlin), 361, 5–25 (in German).

Prandtl, L., 1925. Bericht uber Untersuchungen zur ausgebildeten Turbulenz. *Zeitschrift für angewandte Mathematik und Mechanik,* 5(2), 136–139 (in German).

Rouse, H., 1965. Critical analysis of open channel resistance. *Journal of Hydraulic Engineering,* 91, 1–25.

Straub, L.G.; Silberman, E., and Nelson, H.C., 1958. Open channel flow at small Reynolds numbers. *Transport ASCE,* 123, 685–706.

Strickler, A., 1923. Beitrage zur Frage der Geschwindigkeits formael und der Rauhigkeitszahaeven fur Stronme Kanale und geschlossene Leitungen. *Mitteilungen des eidgenossischen Amtes fur Wasserwirtschaft,* No. 16 (Bern, Switzerland), 265–268 (in German).

Varwick, F., 1945. Zur Fließformel für offene Künstliche Gerinne. Dresden, Germany: Dresden University (in German).

Williams, G.S. and Hazen, A., 1933. *Hydraulic Tables*, 3rd edition. New York: Wiley, 115p.

Yoo, D.H., 1993a. Friction factor for circular pipe with uniform roughness. *Korean Society of Civil Engineers,* 13(5), 165–172.

Yoo, D.H., 1993b. Pipe friction in transition flow. *Korean Society of Civil Engineers,* 13(4), 101–109.

Yoo, D.H. and Lee, M.H., 1999. Logarithmic friction factor equations of open channel flow. *Korean Society of Civil Engineers,* 19(2), 159–167.