A Note on the Nodal Tide in Sea Level Records

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ABSTRACT


This note discusses the likely amplitude and spatial dependence of the nodal (18.61 year) tide in mean sea level (MSL) records. The period of this signal is comparable to the length of many MSL records and so could conceivably affect estimates of secular trends and accelerations. However, it is shown that sufficient knowledge of the nodal tide exists to enable "correction" of the records, should that be required. This approach is likely to yield more reliable estimates of trends from medium-length records than a suggested alternative approach of using nodal terms computed from the individual records.

INTRODUCTION

A recent paper by Houston and Dean (2011) in this journal commented on the desirability of "correcting" long sea level records for the nodal tide to obtain more accurate estimates of trends and accelerations. There has also been interest in the altimeter community on how best to account for the nodal tide (and, in principle, other long-period tides) in sea level time series, now that the record of precise altimetry is two decades long.

I comment on Houston and Dean's paper later, but first it is worth extending their paper slightly to mention a little more of what we know of long-period tides in general and the nodal tide in particular. To do this, I repeat some of the more important calculations on the spatial dependence of the self-consistent equilibrium tide, following Agnew and Farrell (1978).

REVIEW OF SOME OF THE MORE IMPORTANT PAPERS ON THIS SUBJECT

Long-period tides observed in mean sea level (MSL) records have main periods of fortnights, months, seasons, and the lunar "nodal period" of 18.61 years (Pugh, 1987). Cartwright and Tayler (1971) list the main components of the tidal potential, indicating few lines between the annual and the nodal terms (consisting primarily of low-amplitude harmonics of the nodal term and lunar perigean terms). Similarly, there are no lines lower in frequency than the nodal one if we neglect consideration of variations in solar perigee (perihelion), which take place over a cycle of 21,000 years. The observed annual and semiannual tides (Sa and Ssa) contain both astronomical and seasonal (climate) components (Pugh, 1987; Tsimplis and Woodworth, 1994); this paper is concerned with only their astronomical components.

Uncertainties in accounting properly for the shorter-period long-period tides (<12 months) in sea level records are unlikely to significantly affect estimates of interannual and decadal sea level variability and secular trends, even in records as short as 20 years (although uncertainties may affect estimates of standard errors of trends). A similar statement can be made with regard to the pole tide, which is due to variations in the Earth's rotation rather than tidal forcing. That tide has main periods of 12 and 14 months, with amplitudes of 1 to 2 cm at 45° N/S, and is adequately modelled from knowledge of polar motion and with an equilibrium assumption for the ocean response (Desai, 2002).

The nodal tide, with its 18.61-year period, is potentially the most important for consideration, because if it were to be large in amplitude, then its signal in either tide gauge or altimeter data could be misinterpreted as significant ocean decadal variability or even a secular trend and acceleration in shorter records, as Houston and Dean (2011) and earlier papers (e.g., Iz, 2006) explain. Therefore, it is important to have an appreciation of its amplitude and spatial distribution.

We may start with the long-standing belief in the tidal community that as the period of a long-period tide increases, its spatial dependence should become more like that expected from the equilibrium tide. Proudman (1960) argued on general principles that

(1) the constituent whose period is nearly 19 years will certainly follow the equilibrium law...
The Nodal Tide in Sea Level Records

Table 1. Long-period tides.*

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Mf</th>
<th>Mm</th>
<th>Ssa</th>
<th>Sa</th>
<th>Nodal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>13.66 d</td>
<td>27.55 d</td>
<td>6 mo</td>
<td>12 mo</td>
<td>18.61 y</td>
</tr>
<tr>
<td>Amplitude (mm) at the equator (not including the 0.69 factor)</td>
<td>21†</td>
<td>11†</td>
<td>9.75</td>
<td>1.55</td>
<td>8.8</td>
</tr>
<tr>
<td>Peak time at the equator</td>
<td>†</td>
<td>†</td>
<td>Days 81 and 284, i.e., when 2h = 0</td>
<td>Day 2, i.e., when (h – p') = 0</td>
<td>1922.7 ± n × 18.61</td>
</tr>
<tr>
<td>Amplitude (mm) at the poles (not including the 0.69 factor)</td>
<td>42†</td>
<td>22†</td>
<td>19.5</td>
<td>3.1</td>
<td>17.6</td>
</tr>
</tbody>
</table>

* Derived from Cartwright and Edden (1973), Cartwright and Tayler (1971), and Doodson and Warburg (1941).
† A complex dependence of amplitude and phase on N (lunar mean longitude of the ascending node).

(2) the semiannual and annual constituents will probably follow the equilibrium law
(3) the fortnightly and monthly constituents will probably not follow the equilibrium law

By “following the equilibrium law,” Proudman meant a long-period tide with an amplitude simply proportional to 3 sin² latitude – 1 and thus maximum amplitude at the poles, zero amplitude at 35° N/S, out-of-phase between poles and equator, and no zonal dependence. See Cartwright (1999), Pugh (1987), and Rossiter (1962) for some history of research into long-period tides.

If the nodal and other long-period tides had their equilibrium form, then they would have amplitudes and phases as shown in Table 1. The amplitude of the nodal tide is similar to that of Ssa. Each of these amplitudes needs to be multiplied by a solid Earth elastic response factor (diminishing factor or combination of Love numbers) of (1 + k₂ – h₂) or approximately 0.69 to account for the change in potential and elastic response of the solid Earth.

However, consistent with Proudman’s statement, it has been known for years from numerical modelling (e.g., Egbert and Ray, 2003; Mathers and Woodworth, 2001), analysis of altimeter data (e.g., Desai and Wahr, 1995) and altimetry assimilated into numerical models (Kantha, Stewart, and Desai, 1998) that significant departures from equilibrium occur for the fortnightly and monthly long-period tides. Egbert and Ray (2003) can be regarded as producing the definitive paper on Mf; their Figure 1 indicates clearly how consideration of even a schematic ocean can reproduce plausible amplitudes and phases for Mf that depart from equilibrium. Although the departures from equilibrium are less for the monthly than the fortnightly tides, a question remains as to how much of a departure occurs at the lower frequencies of the semiannual, annual, and nodal astronomical tides.

**REVISIT OF AGNEW AND FARRELL (1978)**

It has been known for years that loading and self-attraction modify the simple spatial dependence of any long-period tide that would otherwise “follow the equilibrium law,” as does mass conservation (Agnew and Farrell, 1978). For the purpose of this article, I attempted to reproduce the Agnew and Farrell analysis for a generic long-period tide, in this case using a 1° square grid. (Agnew and Farrell used a grid that was 5° at the equator and denser at higher latitudes; D. Agnew, personal communication.) A Green’s function for loading was used based on the one in Francis and Mazzega (1989) and listed in column 4 of Table 1 of Stepanov and Hughes (2004).

For consistency with Agnew and Farrell (1978), I considered a generic long-period tide with an applied potential (V) as follows:

\[ \frac{V}{g} = 0.69 \times 20(3\sin^2 \text{ latitude} – 1.0), \]  

where g is the acceleration due to gravity and where the 20 can be taken, for illustrative purposes, to be in millimetres. This is a little more than twice the amplitude of the nodal tide (Table 1), so when the nodal tide is discussed, scaling of approximately 44% should be remembered. If this generic tide had the classical equilibrium form of the textbooks, as discussed earlier, then after multiplying by the 0.69 factor and with no regard for loading, there would be a peak amplitude of 27.6 mm at the poles.

However, as Agnew and Farrell discussed (see in particular their equation 2.2), there are two important factors to consider. First, if I use the 1° square grid mentioned previously and compute the ratio of the response of a global ocean (with no land) when loading is included compared to that with no loading (and no land), then the response is almost uniformly 1.24 times the classical one, aside from narrow zonal bands around 35° N/S, where the denominator in this ratio is zero. This value is the about “25%” of the increase in tidal amplitude due to loading compared to the classical result mentioned by Agnew and Farrell (1978, p. 174). (Their equation 2.5 suggests that the same ratio should be obtained everywhere. In the present exercise, I obtained larger values at the highest latitudes, probably due to the use of a coarse square grid.)

Second, we need to consider the role of mass conservation through the cycle due to the presence of land. If I use Agnew and Farrell’s same equation 2.2 and 1° grid, I can compute the response of a realistic ocean area compared to that of the global (no land) ocean, with no loading in each case. In this case, there is a diminution of about 10% compared to the classical response in tropical areas and an increase of 5 to 10% at high latitudes. This analytical finding was confirmed by inspecting the output of a run of a global barotropic numerical model for 19 years with no loading applied to its long-period component.

These two important factors combine to provide (although do not necessarily exactly add to) Figure 1. This shows what Agnew and Farrell called the “self-consistent equilibrium tide,” which accommodates loading and self attraction and mass conservation for a realistic ocean. It is this distribution that I recommend later for use in correcting MSL records.
As a check, Figure 2 shows my version of Figure 1 of Agnew and Farrell (1978). It shows the self-consistent response of the realistic ocean with loading (Figure 1), minus what Agnew and Farrell call the equilibrium tide, with a global ocean (no land) and loading. Figure 2 compares reasonably well to their Figure 1, especially when we consider that different-sized model grids and slightly different Green’s functions were used.

Figure 3a presents the ratio of the self-consistent equilibrium tide (Figure 1) to the classical equilibrium tide of the textbooks (Equation 1). It is a somewhat complicated plot but indicates how wrong someone might be in using the classical form (Equation 1). It can be seen that at higher latitudes, there is a significant enhancement of 25% over the classical value because of the loading discussed earlier. However, in tropical areas, the response obtained is within 5 to 10% of the classical calculation. This is illustrated further in Figure 3b, which shows values of this ratio versus latitude at 180° E/W; we can see from Figure 3a that similar distributions will be obtained at other longitudes. Note the large excursions around 35° N/S due to the division by zero in the denominator at those latitudes.

As a further consistency check, I repeated Figure 1 but with scaling of 100/63.4 so as to obtain a comparable quantity to the “pure ocean tide admittance” in Figure B1 of Ray and Cartwright (1994). The two maps are not exactly the same; there are slightly different negative maxima areas in the tropical Pacific, but otherwise there was decent agreement. As Richard Ray has pointed out (personal communication), a nice thing about this plot (i.e., the scaled version of Figure 1 here or Ray and Cartwright’s Figure B1) is that the amplitude of any long-period tide can be obtained by multiplying the plot’s values as a percentage by the amplitude in the Cartwright-Tayler-Edden tables.

All the preceding discussion applies to the generic long-period tide, which should correspond more to the form of the nodal one for the reasons given previously. However, if dynamical factors come into play, as they do for Mf and Mm (Egbert and Ray, 2003), then there will be additional differences in the real ocean. These differences should be small at the nodal period, but so far as we know, an ocean model has never been run for 19 years with loading to test for any minor residual dynamical signals (as explained later).

**“OBSERVED” NODAL VARIATIONS IN MSL**

A number of papers can be found in the literature that report nodal signals in tide gauge records (see Houston and Dean,
2011, for mention of several). Almost certainly, the majority of these reports are a consequence of misidentified ocean decadal variations. Among notable reliable studies, Rossiter (1967) examined European records, concluding that there was little evidence for a consistent nodal signal. The only tide gauge study that has any plausibility for identifying a nodal signal is that of Trupin and Wahr (1990), who studied a “stack” of records (as in seismic research) rather than individual ones. They concluded that the aggregate nodal signal was consistent with equilibrium expectations. This study was repeated at this laboratory a couple of years ago, now that the Permanent Service for Mean Sea Level (PSMSL) data set is twice as large as the one that Trupin and Wahr analysed and, in particular, contains a number of records from high latitudes in the Arctic. However, similar results were obtained, and the findings were not published. Consequently, I concur with Cartwright (1999), who commented that “it seems unlikely that this conclusion [of Trupin and Wahr] will be seriously challenged in the foreseeable future.”

However, some situations can be envisaged in which significant nonequilibrium nodal signals might occur in sea level records. For example, these might be in shelf areas, where shallow water dynamics somehow generate nodal aliases (cf. chapter 4 of Pugh, 1987). The nonlinear processes that result in departures from equilibrium nodal variations of the diurnal and semidiurnal tides (e.g., departures from the 3.7% nodal variation of $M_2$ amplitude; see, e.g., Woodworth, Shaw, and Blackman, 1991) must have some representation in MSL. The research into such processes requires local modelling and is beyond the general statements of this article.

A second, more obvious situation arises when mean high waters (MHWs) are used as the time series of pseudo-MSL (e.g., Woodworth, Menéndez, and Gehrels, 2011; Wöppelmann et al., 2008). MHW records clearly contain nodal and perigean variations (and possibly their harmonics) of the diurnal and semidiurnal tides that need to be filtered from the records before they can be used in an MSL-like analysis. Annual mean tide level (MTL) is the average of mean high and low waters, and in principle, its time series should correspond to that of MSL with an offset that depends on shallow water effects. Similar comments on nodal contributions to MTL records apply as for MSL records.

### NODAL VARIATIONS IN ALTIMETER DATA

The preceding discussion refers to the pure ocean tide. In studies of satellite radar altimetry, the observed geocentric tide is a combination of the body tide, which is a straightforward scaling of the tidal potential, and the “elastic ocean tide,” which is the ocean tide and its loading.

As far as nodal variation is concerned, its contribution to the body tide will be almost identical to its classical equilibrium form (the $h_2$ component of the scaling of the tidal potential). If the ocean’s response was also the classical one $(1 + k_2 - h_2)$, and
Figure 3. (a) Ratio of the self-consistent equilibrium tide (Figure 1) to the classical equilibrium tide of the textbooks (Equation 1). To avoid crowding of contour lines, contours are drawn every 0.02 between 0.7 and 1.5 only. (b) Values of ratio versus latitude at 180° E/W.
if it did not load the solid Earth, then the altimeter would see a combined signal \((1 + k^2)\) times the potential. The diurnal and semidiurnal tides clearly do not have this classical equilibrium response, so a loading calculation is essential for them. However, my discussion of Agnew and Farrell (1978) has shown that the spatial variation of the nodal tide departs from its classical form in Equation 1 as well. Consequently, in principle, loading also must be considered in this case.

I can say “in principle,” because it is clear that any loading in this case is only at the millimetre level. This can be readily computed with the use of a Green’s function such as in column 2 of Table 1 of Francis and Mazzega (1990). Figure 4 shows the loading corresponding to the self-consistent long-period tide of Figure 1.

Now that altimetry has provided two decades of high-quality data, people have started to investigate whether possible nodal signals in that data set have an equilibrium response. Cherniawsky et al. (2010) searched for nodal signals in altimeter spatial fields but concluded that the obtained “amplitudes and phase values likely arise from this [ocean] variability and not the nodal tide.”

As far as quasiglobal averages of MSL from altimetry are concerned (i.e., \(\pm 66^\circ\) N/S for TOPEX/Poseidon and Jason), it can be seen from Figure 1 that any nodal contribution will be minimal, at least for timescales of decades. Over a few years and in regional, rather than global, averages, we might imagine errors on the order of 2 mm/y being introduced by omission of a nodal signal, such as a range \((2 \times \text{amplitude})\) of roughly \(2 \times 6\) mm over 6 years, and perhaps double that at the poles. Once records span a couple of decades or so, this problem also disappears.

**RECOMMENDATIONS FOR CORRECTING SEA LEVEL RECORDS**

Inferences that the ocean’s response at the nodal period cannot be far from equilibrium can be obtained from other literature. For example, tests for mantle anelasticity from geodetic observations appear to be at least consistent with an equilibrium response for both pole and nodal tides (Benjamin et al., 2006). Desai (2002) measured the pole tide to be within 3% of equilibrium in its self-consistent form; see also Dickman (1985) and Carton and Wahr (1986).

This brings us back to Houston and Dean (2011). Part of that paper is an interesting discussion on how omitting consideration of an integral number of periods of a long-period tide in a record, or omitting consideration of it completely, could distort an estimate of underlying trend and acceleration (see also Iz,
However, it is a somewhat numerically idealistic discussion (e.g., it omits consideration of additional non-tidal low-frequency variability, which in a realistic analysis cannot be treated as white noise). The authors also suggest how, in studies of trends and accelerations in medium-length records, an additional periodic term of 18.61 years could be added to a regression, with nodal tide amplitude and phase determined in the fit. (The authors do not appear to have considered a variant of their analysis: to constrain the phase of the nodal term to that of the equilibrium tide and allow only the amplitude to vary.) Their text is replete with expressions like “accounting for the nodal tide” when they mean “accounting for that part of the variance at 18.61 years,” and it is clear that at most locations the ocean variability around this period is likely to be an order of magnitude or more larger than the real nodal tide.

The second part of Houston and Dean (2011) clearly makes the case (as did, e.g., Rossiter, 1967) that such a low-amplitude, low-frequency tide is not resolvable, at least at most locations, by a form of harmonic analysis of an individual record with a length of several decades or even a century. Therefore, there is no logic in trying to estimate it empirically in a regression and thereby correct for it. It seems that if the Houston and Dean’s suggestion was to be followed, then we are just as likely to obtain less, rather than more, reliable trends and accelerations.

At present, the only logical approach to correct for the nodal tide at most locations, given that we know it has an equilibrium form or one close to it, is to subtract the self-consistent values of Figure 1 or, if that is too complicated, to subtract the simple Equation 1 (both appropriately scaled). The largest mistake we could make in using Equation 1 would be about 25%, or just 1 to 2 mm for the nodal tide at most locations. This suggestion applies to records of any length. Houston and Dean (2011) demonstrated that for record lengths more than about 60 years, consideration of an 18.61-year signal has little impact on determined trends and accelerations; for shorter ones, the general ocean variability will dominate any discussion, as Douglas (1992) and others showed long ago. (For a more recent and graphic illustration of the importance of “decadal” ocean variability in terms of the number of years required to obtain a given standard error on a measured trend in different parts of the ocean, see Hughes and Williams, 2011.) Nevertheless, the nodal correction shown earlier is straightforward to apply to the longer records, if desired (although, as inferred from Figure 1, I am tempted to suggest just forgetting it for many midlatitude coastlines).

WAYS FORWARD

How can we take this topic forward? First, to pin down how far from equilibrium the nodal tide is, and to test for small dynamical effects at 18.61 years, one desirable task would be to run a barotropic model for 19 years with explicit loading. To my knowledge, such a computationally intensive activity has never been undertaken, although it is conceptually straightforward. Spectral tidal modelling may be more efficient.

Second, Richard Ray and Trevor Baker have pointed out (personal communication) that, aside from considering whether the dynamical effects are inconsequential (presumably Proudman was essentially correct about that), there is an uncertainty in calculating the nodal tide due to uncertainties in mantle anelastic effects. At the long periods discussed here, the Love numbers change from the $h_2$ and $k_2$ of the textbooks and become complex, inducing a small out-of-phase component. In Benjamin et al. (2006), the authors did allow for that possibility: the uncertainty in Love numbers at 18.61 years resulting in the wide spread of curves in Figures 6 and 7 of that paper. Further research along these lines would be valuable.

Finally, and more likely in the short term, innovative analyses of the PSMSL data set may be developed, possibly using data in some stacked form, as did Trupin and Wahr (1990), to test more rigorously for consistency with equilibrium globally and regionally. Even though we failed to improve on the Trupin and Wahr analysis, even with twice the data that they had at their disposal, this result should not stop others from trying.

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