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Unsaturated Flow Characterization Utilizing Water Content Data Collected within the Capillary Fringe

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ABSTRACT: An analysis is presented to determine unsaturated zone hydraulic parameters based on detailed water content profiles, which can be readily acquired during hydrological investigations. Core samples taken through the unsaturated zone allow for the acquisition of gravimetrically determined water content data as a function of elevation at 3 inch intervals. This dense spacing of data provides several measurements of the water content within the capillary fringe, which are utilized to determine capillary pressure function parameters via least-squares calibration. The water content data collected above the capillary fringe are used to calculate dimensionless flow as a function of elevation providing a snapshot characterization of flow through the unsaturated zone. The water content at a flow stagnation point provides an in situ estimate of specific yield. In situ determinations of capillary pressure function parameters utilizing this method, together with particle-size distributions, can provide a valuable supplement to data libraries of unsaturated zone hydraulic parameters. The method is illustrated using data collected from plots within an agricultural research facility in Wisconsin.

KEYWORDS: unsaturated zone, unsaturated flow, specific yield, capillary pressure functions

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Introduction

The modeling of flow and water storage in the unsaturated zone is complicated by the non-linearity and complexity of the basic hydraulic functions: capillary pressure and hydraulic conductivity.[1,2] Another challenge encountered in quantifying recharge and the transport of compounds for a particular setting is obtaining site-specific characterizations of hydraulic functions. The extensive field installations and laboratory testing needed to determine these functions at depth are time consuming and beyond the scope of typical hydrogeologic site investigations. Pedotransfer functions (PTFs) can be used to provide estimates of the parameter values of the hydraulic functions[3–5] by relating basic soil data such as particle-size distributions, bulk density, and organic carbon content to libraries of hydraulic parameter values such as the US Department of Agriculture database Rosetta.[6] Particle-size and organic carbon data obtained from the sub-samples of sediment cores provide reliable input parameters for PTFs, but the disturbance associated with coring and repacking samples adds uncertainly to determinations of bulk density. Established methods exist for collecting undisturbed cores of saturated or nearly saturated unconsolidated sediments,[7,8] cohesive sediments,[9] and soils.[10] Users of PTFs are limited to reference values or estimates of bulk density based on particle-size distributions until equivalent methods are developed for the collection of undisturbed cores within unsaturated, non-cohesive sediments.

A method to determine capillary pressure function parameters based on the water content data obtained within the capillary fringe, the layer in which the groundwater seeps upward from the water table, is presented. The water content can be readily acquired from the core samples collected during the observation of well installations or other common hydrogeologic site characterization tasks, and when supplemented with particle-size distribution measurements, dimensionless flow as a function of distance above the water table can be calculated providing a snapshot characterization of unsaturated zone flow conditions and an in situ estimate of specific yield (S_y), the quantity of water a unit volume of aquifer material will provide by gravity drainage. The application of this method is illustrated using samples collected from a sandy, non-cohesive unsaturated zone underlying an agricultural research facility in Wisconsin.

Unsaturated Flow Model

Darcy's law for one-dimensional flow in the unsaturated zone is as follows:

$$q = -K \frac{\mathrm{d}h}{\mathrm{d}z} \tag{1}$$

where q is the specific discharge (cm/s); K is the unsaturated conductivity (cm/s); z is the vertical coordinate, positive upward (cm); and h is the head (cm) defined as follows:

$$h = z - \psi \tag{2}$$

where ψ is the pressure head or capillary pressure function, which is defined here as a positive function (cm). Combining equations (1) and (2) gives the unsaturated flow equation:

$$q = -K\left(1 - \frac{\mathrm{d}\psi}{\mathrm{d}z}\right) \tag{3}$$

The van Genuchten model[2] for ψ in terms of the water content is as follows:

$$\psi = \frac{1}{\alpha}\left(S_e^{\,n/(1-n)} - 1\right)^{1/n} \tag{4}$$

where α (1/cm) and n (dimensionless) are fitted parameters of the van Genuchten model. Relative saturation (S_e) is defined as follows:

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{5}$$

where θ is the water content (cm^3/cm^3), and θ_r (cm^3/cm^3) and θ_s (cm^3/cm^3) are the residual and saturated water contents, respectively. Equation (4) is used in conjunction with the pore-size distribution model by Mualem[11] to yield the van Genuchten–Mualem equation for the determination of K[5]:

$$K = K_o (S_e)^L \left\{1 - \left[1 - (S_e)^{n/(n-1)}\right]^{(1-1/n)2}\right\} \tag{6}$$

where K_o is the conductivity at saturation (cm/s) and L (dimensionless) are fitted parameters. PTFs, like Rosetta,[5] allow for estimating the values of the parameters based on the sediment textural data (percentage of sand, silt, and clay).

Calibration of the van Genuchten model with the capillary fringe water content data. The van Genuchten model (equation (4)) is an empirical model with fitted parameters α and n. Although the water content parameters θ_r and θ_s have physical interpretation, these parameters may also be viewed as fitted parameters.

Equation (3) is rearranged to obtain an expression for ψ:

$$\frac{\mathrm{d}\psi}{\mathrm{d}z} = 1 + \frac{q}{K} \tag{7}$$

For the water content typical of the capillary fringe ($0.3 < S_e < 1$), the values for K are expected to be much greater than the magnitude of q, therefore, within the capillary fringe:

$$\frac{\mathrm{d}\psi}{\mathrm{d}z} \approx 1 \quad \text{or} \quad \psi \approx z \tag{8}$$

where the water table elevation is the datum $z = 0$. This assumption is only for the capillary fringe.

The validity of this assumption (equation (8)) can be illustrated by determining $K(S_e)$ for $0.3 < S_e < 1$ via equation (6) and utilizing average parameter values of sand from the USDS database Rosetta (K_o = 24.5 cm/day, L = −0.93, n = 3.18, θ_s = 0.375, and θ_r = 0.053).[6] Typical humid or irrigated recharge rates (q = 25 cm/year) are at least an order of magnitude less than K for values $S_e > 0.3$ (points within the capillary fringe).

Equation (4) is rearranged and combined with equation (5) and approximation (8) to give an expression of the water content in the capillary fringe:

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{\left[1 + (\alpha z)^n\right]^{1 - 1/n}} \tag{9}$$

Each water content data point collected within the capillary fringe yields an equation of the form (9). Multiple data points result in a system of equations that is solved to obtain estimates for the parameters α, n, θ_r, and θ_s.

The water content data are obtained as follows:

$$\theta = \rho_b w \tag{10}$$

where w is the volume of water in the sediment sample (mass of water multiplied by water density, assumed to be equal to 1 g/cm^3) and ρ_b is the dry bulk density of the sediment sample:

$$\rho_b = \gamma(1-\theta_s) \tag{11}$$

where γ is the grain density (eg 2.62 g/cm^3 is the density of quartz). Here the physical interpretation of θ_s as the saturated water content (porosity) is used to define ρ_b.

Dimensionless flow in the unsaturated zone. The water content data above the capillary fringe allows for estimating dimensionless flow, q/K. Equation (3) is rearranged to give:

$$\frac{q}{K} = -\left(1-\frac{\mathrm{d}\psi}{\mathrm{d}z}\right) \tag{12}$$

The derivative dψ/dz is approximated by finite difference approximations with ψ defined for each water content data point (equation (4)). Given the selected sign convention, $q/K < 0$ indicates the downward flow. In particular, $q/K = -1$ corresponds to gravity-driven flow with negligible capillary forces (unit gradient conditions). The upward flow is indicated if $q/K > 0$, which occurs when the upward capillary force exceeds the downward force of gravity (wicking conditions). These calculations assume that the parameters defining ψ are constant and equal to those of the sediment of the capillary fringe; therefore, this analysis can deteriorate with the distance above the capillary fringe as the sediment properties can change. This assumption is reasonable in the systems where the texture and bulk density of the capillary zone is similar to those of the overlying sediments.

When $q/K = 0$, downward gravity and upward capillary forces are balanced and a field-derived estimate for S_y is obtained at that location:

$$S_y = \theta_s - \theta \tag{13}$$

As determined by this method, S_y is dependent on the prevailing water content distribution and is not a static property of the unsaturated sediment as it is commonly defined. Field-determined and site-specific estimates for S_y would improve estimates of recharge obtained by the analysis of time series water table elevation data described by Healy and Cook.[12]

Example model application—Hancock, Wisconsin. The model application is demonstrated by the evaluation of moisture content data collected at four locations as part of a groundwater quality study conducted on potato and cucumber plots post-harvest at the Hancock Agricultural Research Station in Hancock, Wisconsin in October 2011. Temporary wells were installed and sediment cores were collected using a Geoprobe® direct push system, Geoprobe SP-16 groundwater sampling system, and Geoprobe dual tube soil sampling system. Water levels were determined using a steel tape. The gravimetric water content of the sediment samples was determined on ~60 g sediment samples collected every 0.25 ft using the methods previously described by Reilly and Baehr.[13] The particle-size distribution of the selected samples was determined by optical diffraction using the Beckman Coulter LS-230 (Brea, CA) particle-size analyzer using methods described in Gee and Or.[14] Visual examination and particle-size measurements indicate that all sediments are loamy sand or sand. Precipitation and irrigation records maintained by the staff of the research station indicate that 74.5 cm of precipitation fell on the farm in the year preceding this study and the selected plots (C21, C10, S25, and E14) received 36.8–44.2 cm/year of irrigation via sprinklers.

The results of model calibration of the water content data for sites C21, C10, S25, and E14 are shown in Figure 1. The gaps in the water content data are due to incomplete core recovery. The red data points in the capillary fringe were used to obtain the least-squares error parameter estimates listed in Table 1. This method assumes the capillary fringe water content has equilibrated with the measured water table elevation. The parameter estimates, therefore, only apply for the sediment layer containing the capillary fringe. Variations in the water content can be due to wetting fronts and grain size changes. The dashed portion of the fitted curves extending above the capillary fringe is the theoretical hydrostatic water content distribution as $q = 0$ was assumed to obtain the fitted parameters. To provide context for the values obtained, parameter values for sands from the Rosetta database[6] are listed. The S_y values reported in Table 1 were obtained using the water content of the lowermost point in the unsaturated zone for which $q/K = 0$.

The plots of q/K are shown in Figure 2 for each site along with the reference conditions (dashed lines) $q/K = 0$ (no flow) and $q/K = -1$ (gravity-dominated flow). For sites C10 and E14, the flow is gravity dominated. The average value for q/K in the lowermost interval above the capillary fringe is −0.95 for both sites indicating gravity-dominated recharge. The flow at sites C10 and E14 is expected to be higher in magnitude than that at sites C21 and S25 because it is gravity dominated. This expectation is also supported by the fact that all the water content data points lie to the right of the extended hydrostatic curve (Fig. 1) for these sites. At sites C21 and S25, portions of the unsaturated zone had predicted upward flow ($q/K > 0$) caused by the capillary forces and the overlying drier water content.

The lowermost interval over which the contiguous water content data exist is selected to estimate the magnitude of flow, q. Additional parameters L and K_o are obtained from the Rosetta database (Table 1) to estimate K via equation (6). For the sites C10 and E14, the average values for q over the lowermost interval above the capillary fringe are −137.9 and −67.1 cm/year, respectively, whereas the average q values for the sites C21 and S25 are lower (as expected) at −23.5

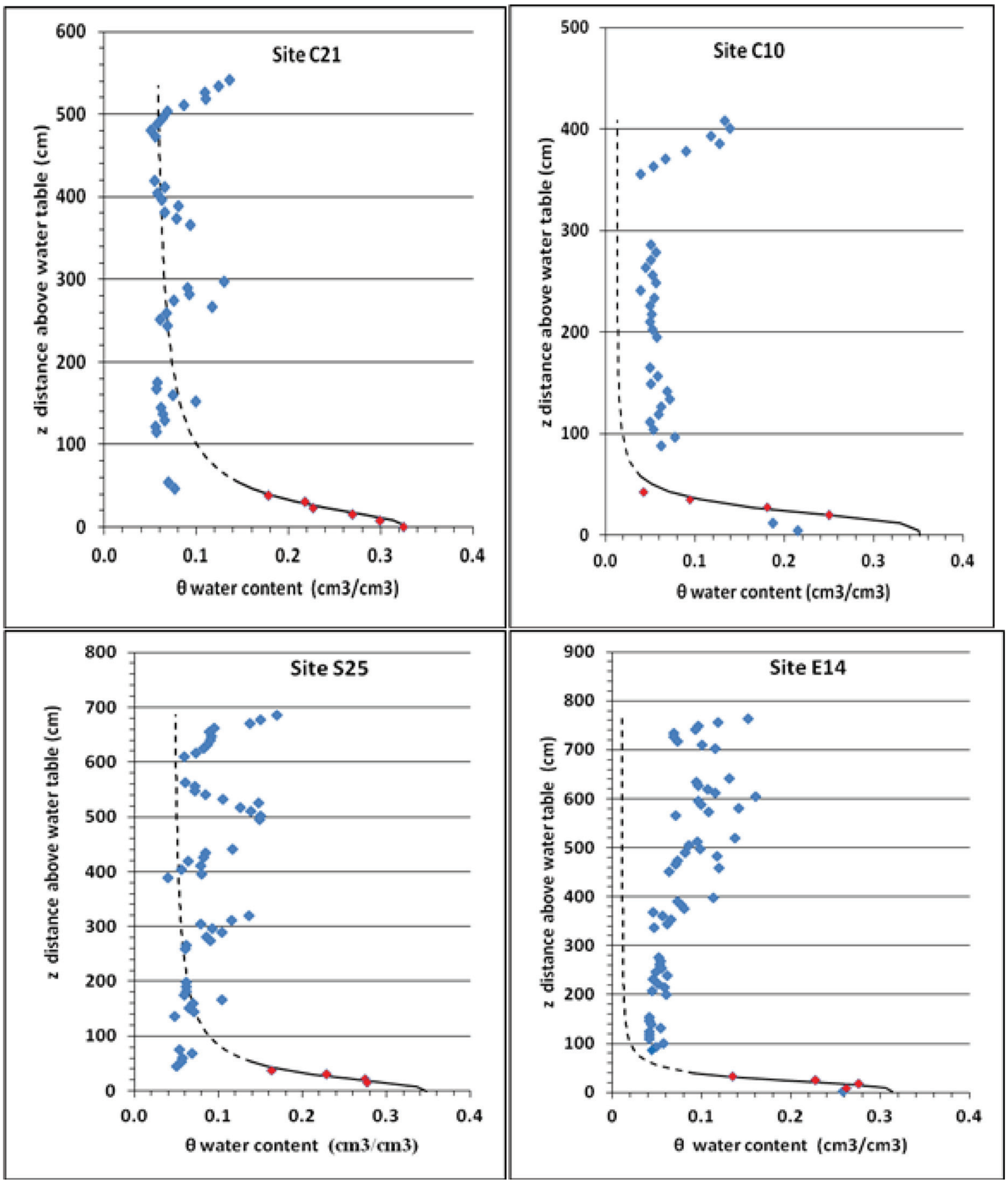

Figure 1. Calibration of the van Genuchten model with the capillary fringe water content data at four selected field sites in Wisconsin. Red data points in the capillary fringe were used to obtain the least-squares error parameter estimates. The dashed portion of the fitted curves extending above the capillary fringe is the theoretical hydrostatic water content distribution.

Table 1. Results of model calibration of the van Genuchten model with the field-derived water content data.

CAPILLARY FRINGE PARAMETERS								CALCULATIONS		
SITE	θ_r (cm^3/cm^3)	θ_s (cm^3/cm^3)	n (D-LESS)	α (1/cm)	S_y (cm^3/cm^3)	K_0 (cm/day)	L (D-LESS)	INTERVAL ABOVE WATER TABLE	AVERAGE q/K (DIMENSIONLESS)	AVERAGE q (cm/yr)
C21	.051	.328	2.14	.045	.267			115–176 cm	–1.53	–23.5
C10	.013	.351	3.72	.044	.246			88–165 cm	–0.95	–137.9
S25	.045	.347	2.29	.044	.259			137–198 cm	–5.8	–4.7
E14	.011	.313	3.47	.041	.190			86–154 cm	–0.95	–67.1
Average	.030	.335	2.91	.044	.241					
Rosetta for Sand	.053	.375	3.18	.035		24.5	–.93			

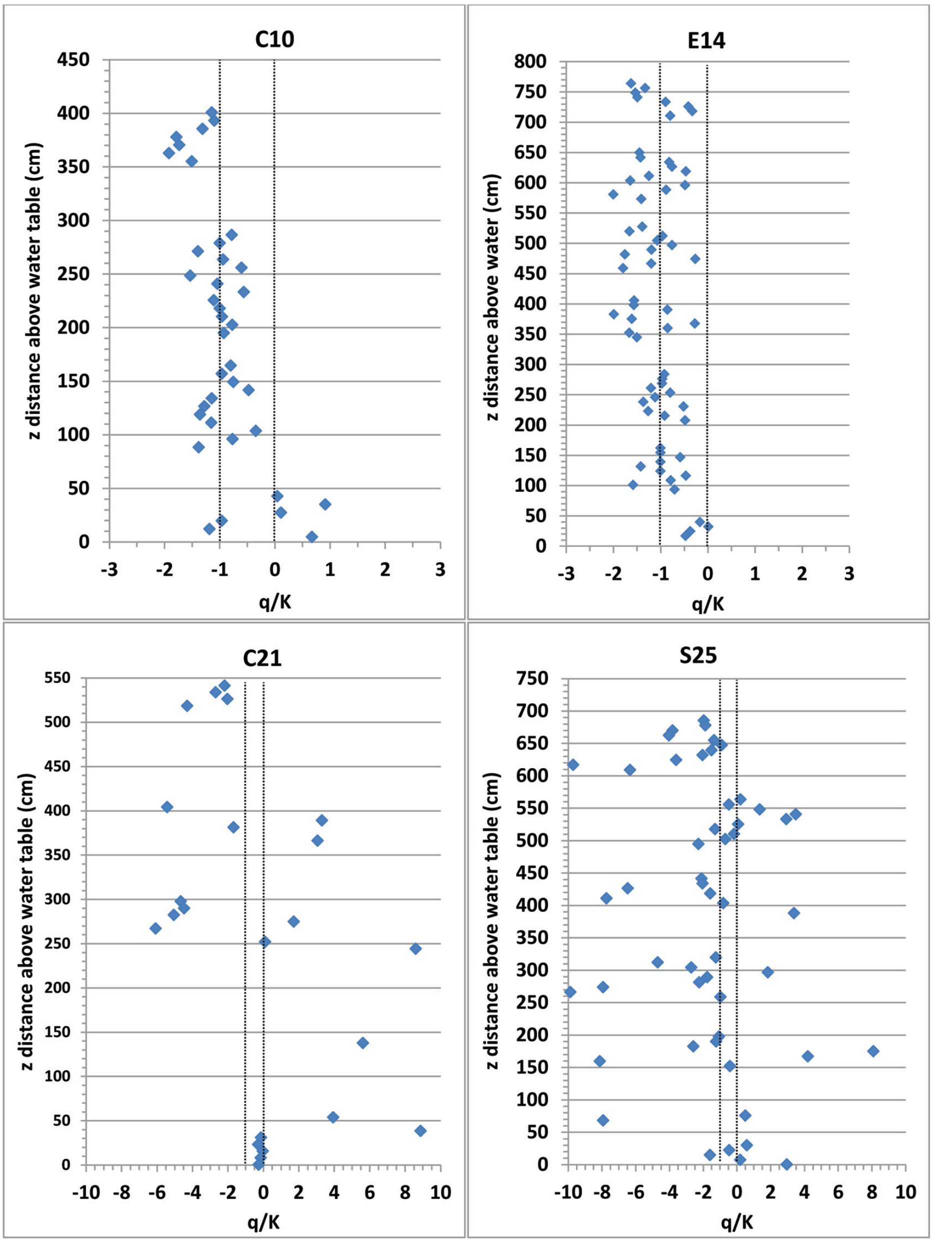

Figure 2. Dimensionless flow (*q*/*K*) at four selected field sites in Wisconsin.

and −4.7 cm/year, respectively. Particle-size analysis of sediments indicates a higher percentage of gravels in E14 and C10 relative to C21 and S25, which may explain the flow differences.

Summary

A field method to determine the capillary pressure function based on the water content data collected within the capillary fringe has been presented and demonstrated. As the first contribution, this method can be applied to supplement unsaturated hydraulic databases such as Rosetta[6] simply by collecting the water content data in conjunction with well installations. Additionally, a snapshot of unsaturated flow conditions can be obtained by collecting the water content data above the capillary fringe, which allows for pointwise determination of capillary pressure and the subsequent pointwise determination of dimensionless flow *q*/*K* as a function of distance above the water table via finite difference approximation.

Although dimensionless flow does not provide actual flow without further efforts or assumptions regarding the unsaturated conductivity, *q*/*K* plotted above the water table provides insight into the variability of recharge that occurs at the spatial scale of an investigation. Recharge is traditionally evaluated at the watershed scale using the base flow stream data, but for the groundwater contamination investigations, the variability of recharge along with differing land use within the study area both contribute to spatially variable chemical transport through the unsaturated zone and loading to the surficial aquifers.

Author Contributions

Conceived and designed the experiments: AB and TR. Analyzed the data: AB and TR. Wrote the first draft of the manuscript: AB and TR. Contributed to the writing of the manuscript: AB and TR. Agree with manuscript results and conclusions: AB and TR. Jointly developed the structure and arguments for the paper: AB and TR. Made critical revisions and approved final version: AB and TR. All authors reviewed and approved of the final manuscript.

DISCLOSURES AND ETHICS

As a requirement of publication the authors have provided signed confirmation of their compliance with ethical and legal obligations including but not limited to compliance with ICMJE authorship and competing interests guidelines, that the article is neither under consideration for publication nor published elsewhere, of their compliance with legal and ethical guidelines concerning human and animal research participants (if applicable), and that permission has been obtained for reproduction of any copyrighted material. This article was subject to blind, independent, expert peer review. The reviewers reported no competing interests.

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