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Improvement of Decomposing Results of Empirical Mode Decomposition and its Variations for Sea-level Records Analysis

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ABSTRACT



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The performance of empirical mode decomposition (EMD) and its variations such as ensemble EMD (EEMD), complete EEMD with adaptive noise (CEEMDAN) and improved CEEMDAN (impCEEMDAN) are tested using artificial signal tests. In the artificial signal test, intrinsic mode functions (IMFs) are obtained using EMD, EEMD, CEEMDAN, and impCEEMDAN and then compared to prescribed oscillations in an artificial a priori known signal. In all cases, extra and redundant modes are found due to residual noises. Furthermore, the low frequency modes are generally distorted. To overcome this problem a novel approach for reconstructing IMFs is proposed, where low-energy redundant modes are merged to one common signal based on statistical significance tests by comparing the energy-density of IMFs with energy-density spread function of white noise with similar scale. Artificial signal tests illustrate that the mode reconstruction method works well in approximating the prescribed true modes. Overall, the impCEEMDAN performs best with a reasonable fit to the original components and statistically significant low-frequency modes. The mode reconstruction method can improve the decomposing and filtering capacity of EMD and its variations.

ADDITIONAL INDEX WORDS: *empirical mode decomposition, mode reconstruction, trend, detrending, sea-level*

INTRODUCTION

For coastal engineering and management purposes sea-level rise is the key factor to be considered because of its large impacts in a changing climate. Therefore, the estimation of long-term trends and changing rates of sea-level records is of eminent importance. Visser *et al.* (2015) pointed that misunderstanding and controversies on acceleration or deceleration of sea-level records are due to mathematical or statistical characteristics of the models in use. The authors reviewed 30 methods applied to sea-level records and made recommendations for good modelling practices. However, while more and more in use to analyze sea-level records (Breaker and Ruzmaikin, 2011; Ezer *et al.*, 2013; Ezer and Corlett, 2012; Lee, 2013; Lee and Kaneko, 2015), empirical mode decomposition (EMD) is not directly assessed in their model evaluation (Visser *et al.*, 2015). In the meantime, Chambers (2015) investigated and mentioned that since the EMD is a purely mathematical deconstruction of data with no regard to the intrinsic covariance of signals or physics behind and a zero mean assumption nearby peaks, it is unlikely that a single intrinsic mode function (IMF) from the EMD analysis can represent a real physical climate variation. Therefore, it is more likely that multiple modes are required to quantify real signals. However, in

case of needing more than one IMF to describe a real physical signal, the question naturally arises, which IMFs should be combined without any a priori knowledge of the real signal.

Here to address this important question, EMD and recent advancements in EMD variations, such as ensemble EMD (EEMD), complete EEMD with adaptive noise (CEEMDAN) and improved CEEMDAN (impCEEMDAN), are applied to a randomly simulated artificial data set where the signal is *a priori* known. Moreover, we introduce a novel approach for reconstructing multiple IMFs into one combined signal based on statistical Monte-Carlo simulations for energy density levels of decomposed IMFs.

METHODS

Empirical mode decomposition (EMD)

The EMD is an adaptive method to decompose a signal $x(t)$ into a number of IMFs, which become the basis representing the signal. The algorithm can be described as follows (Colominas *et al.*, 2014; Huang *et al.*, 1998):

Step 1. Set the IMF index $k = 0$ and find all extrema of the 0 th residue $r_0 = x$.

Step 2. Interpolate between minima (maxima) of r_k to obtain the lower (upper) envelope e_{min} (e_{max}).

Step 3. Compute the mean envelope $m = (e_{min} + e_{max})/2$.

Step 4. Compute the IMF candidate $d_{k+1} = r_k - m$.

Step 5. Is d_{k+1} an IMF?

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Yes. Save d_{k+1} , compute the residue $r_{k+1} = x - \sum_{i=1}^k d_i$, do $k = k + 1$, and treat r_k as input data in step 2.

No. Treat d_{k+1} as input data in step 2.

Step 6. Continue until the final residue r_k satisfies some predefined stopping criterion.

The refinement process (steps 2 to 5) is needed to extract every mode by a certain number of iterations and is named as *sifting* process. EMD is ideally suited for analysing data from non-stationary and non-linear processes. However, EMD still cannot resolve the most complicated cases, when the processes are non-linear and the noises also have the same time-scale as the signal; their separation becomes impossible (*mode mixing*).

Ensemble empirical mode decomposition (EEMD)

The EEMD defines the “true” modes (here noted as $\overline{IMF} = \bar{d}$ in what follows) as the average of the corresponding IMFs obtained from an ensemble of the original signal, x , plus different realizations of finite variance white noise. The EEMD algorithm can be described as follows (Colominas *et al.*, 2012; Wu and Huang, 2009):

Step 1. Generate $x^{(i)} = x + \beta w^{(i)}$, where $w^{(i)}$ ($i=1, \dots, I$) are different realization of zero mean unit variance white noise, I is the number of realization in the ensemble and the magnitude of added noise $\beta > 0$.

Step 2. Decompose completely each $x^{(i)}$ ($i=1, \dots, I$) by EMD, obtaining the modes $d_k^{(i)}$, where $k = 1, \dots, K$ indicates the mode.

Step 3. Assign \bar{d}_k as the k th mode of x , obtained by averaging the corresponding modes: $\bar{d}_k = \frac{1}{I} \sum_{i=1}^I d_k^{(i)}$.

The extraction of every $d_k^{(i)}$ requires a different number of sifting iterations. It can be noticed that in EEMD, every $x^{(i)}$ is decomposed independently from the other realizations and for every one of them a residue $r_k^{(i)} = r_{k-1}^{(i)} - d_k^{(i)}$ is obtained at each stage, with no connection between the different realizations. This situation is the cause of some EEMD disadvantages: (i) the decomposition is not complete and (ii) different realizations of signal plus noise might produce different number of modes particularly in low frequency.

Complete EEMD with adaptive noise (CEEMDAN)

To improve these drawbacks, a new ensemble method called CEEMDAN was proposed (Colominas *et al.*, 2012; Torres *et al.*, 2011). The general idea is the following: $x^{(i)}$ are generated from x and the first mode $\bar{d}_1 = \bar{d}_1$ is computed exactly as in EEMD. Then, a unique first residue is obtained, independently from the noise realization:

$$r_1 = x - \bar{d}_1 \quad (1)$$

After that, the first EMD mode is computed from an ensemble of r_1 plus different realizations of a particular noise. The second mode \bar{d}_2 is defined as the average of these modes. The next residue is: $r_2 = r_1 - \bar{d}_2$. This procedure continues until a stopping criterion is reached.

The next algorithm details the CEEMDAN method. Let $E_k(\cdot)$ be the operator which produces the k th mode obtained by EMD and let $w^{(i)}$ be a realization of zero mean unit variance white noise. Then:

Step 1. For every $i=1, \dots, I$ decompose each $x^{(i)} = x + \beta_0 w^{(i)}$ by EMD, until its first mode and compute

$$\bar{d}_1 = \frac{1}{I} \sum_{i=1}^I d_1^{(i)} = \bar{d}_1 \quad (2)$$

Step 2. At the first stage ($k = 1$) calculate the first residue as in Eq. (1): $r_1 = x - \bar{d}_1$.

Step 3. Obtain the first mode of $r_1 + \beta_1 E_1(w^{(i)})$, $i=1, \dots, I$ by EMD and define the second CEEMDAN mode as:

$$\bar{d}_2 = \frac{1}{I} \sum_{i=1}^I E_1(r_1 + \beta_1 E_1(w^{(i)})) \quad (3)$$

Step 4. For $k = 2, \dots, K$ calculate the k th residue:

$$r_k = r_{(k-1)} - \bar{d}_k \quad (4)$$

Step 5. Obtain the first mode of $r_k + \beta_k E_k(w^{(i)})$, $i=1, \dots, I$ by EMD until define the $(k + 1)$ th CEEMDAN mode as:

$$\bar{d}_{(k+1)} = \frac{1}{I} \sum_{i=1}^I E_1(r_k + \beta_k E_k(w^{(i)})) \quad (5)$$

Step 6. Go to step 4 for the next k .

Iterate the steps 4 to 6 until the obtained residue cannot be further decomposed by EMD, either because it satisfies IMF conditions or because it has less than three local extrema.

Observe that, by construction of CEEMDAN, the final residue fulfills:

$$r_k = x - \sum_{k=1}^K \bar{d}_k \quad (6)$$

with K being the total number of modes. Therefore, the signal of interest x can be expressed as

$$x = \sum_{k=1}^K \bar{d}_k + r_k \quad (7)$$

ensuring the completeness property of the proposed decomposition and thus providing an exact reconstruction of the original data. The final number of modes is determined only by the data and the stopping criterion. The coefficient $\beta_k = \varepsilon_k \text{std}(r_k)$ allow the selection of the SNR at each stage, where ε is the noise standard deviation.

However, CEEMDAN still has some aspects that have to be improved further: (i) its modes contain some residual noise; and (ii) the signal information appears “later” than in EEMD with some “spurious” modes in the early stages of the decomposition.

Let us recall the operator $E_k(\cdot)$, and let $M(\cdot)$ be the operator which produces the local mean of the signal that is applied to. It can be noticed that $E_1(x) = x - M(x)$. Let $w^{(i)}$ be a realization of white Gaussian noise, $x^{(i)} = x + w^{(i)}$, and $\langle \cdot \rangle$ the action of averaging throughout the realizations. For the first EEMD and original CEEMDAN modes we have:

$$\bar{d}_1 = \langle E_1(x^{(i)}) \rangle = \langle x^{(i)} - M(x^{(i)}) \rangle = \langle x^{(i)} \rangle - \langle M(x^{(i)}) \rangle \quad (8)$$

By estimating only the local mean and subtracting it from the original signal, we have:

$$\bar{d}_1 = x - \langle M(x_i) \rangle \quad (9)$$

In this way, we obtain a reduction in the amount of noise present in the modes.

Improved CEEMDAN

Taking into account the two drawbacks of CEEMDAN, an improved algorithm for CEEMDAN (hereinafter, impCEEMDAN) is proposed as follows (Colominas *et al.*, 2014):

Step 1. Calculate by EMD the local means of I realizations $x^{(i)} = x + \beta_0 E_1(w^{(i)})$ to obtain the first residue

$$r_1 = \langle M_1(w^{(i)}) \rangle \quad (10)$$

Step 2. At the first stage ($k = 1$) calculate the first mode: $\bar{d}_1 = x - r_1$.

Step 3. Estimate the second residue as the average of local means of the realizations $r_1 + \beta_1 E_2(w^{(l)})$ and define the second mode:

$$\tilde{d}_2 = r_1 - r_2 = r_1 - \langle M(r_1 + \beta_1 E_2(w^{(l)})) \rangle \quad (11)$$

Step 4. For $k = 3, \dots, K$ calculate the k th residue

$$r_k = \langle M(r_{k-1} + \beta_{k-1} E_k(w^{(l)})) \rangle \quad (12)$$

Step 5. Compute the k th mode

$$\tilde{d}_k = r_{k-1} - r_k \quad (13)$$

Step 6. Go to step 4 for next k .

Constants $\beta_k = \varepsilon_k \text{std}(r_k)$ are chosen to obtain a desired SNR between the added noise and the residue to which the noise is added. In order to obtain noise realizations with smaller amplitudes for the late stages of the decomposition, in the rest of the modes we will use the noise as resulting from its pre-processing by EMD, i.e., without normalizing them by its standard deviation ($\beta_0 = \varepsilon_0 \text{std}(r_k), k \geq 1$). In this study, $\varepsilon = 0.02, l = 500$, a few hundred of realization, and the same SNR are used for all the stages in all analysis.

Statistical significance test

The ability of EMD to obtain the best IMF with statistical significance, physical meaning and uniqueness is debatable. To determine whether a dataset or its components contain useful information, statistical significance tests were performed using Gaussian white noise with EEMD, following Wu and Huang (2004). First, decompose the targeted noisy dataset (normalized) into IMFs; second, calculate the spread function of given percentiles for the IMFs decomposed from Gaussian white noise; third, select the confidence level (e.g., 95%) and determine the upper and lower spread lines; and finally, compare the energy density for the IMFs from the noisy data with the spread functions. The IMFs with energy located above the upper bound or below the lower bound contain signal information at the selected confidence level. In the final step, if the targeted dataset is non-stationary and it has a significant trend, then the trend should be excluded because the Gaussian white noise does not contain any trend; then, the energy densities of all oscillatory IMFs should be rescaled according to the total energy of all the oscillatory IMFs

before comparing the energy densities of the IMFs with the spread functions from the Gaussian white noise.

Mode reconstruction

Here, we propose a new approach to reconstruct IMFs to increase their statistical significances by using the result of statistical significance test. In nature, an average period of an IMF is approximately two times longer or two times shorter than neighboring IMFs (Huang et al., 1998). Therefore, we propose to reconstruct modes if an IMF does not fulfill one of following conditions: (i) an average period of an IMF is attributed to be approximately two times longer or two times shorter than the neighboring IMF, (ii) an IMF should be statistically significant with an energy level of an IMF over 95% confidence level.

If an IMF is statistically insignificant due to low-energy level, or its average period is not twice longer or shorter than neighboring IMFs, then we can combine the insignificant IMF with a statistically significant neighboring IMF. After combining those insignificant IMFs to a common signal, a new set of IMFs can be obtained with all resulting modes significant over 95% confidence level.

EXPERIMENTS

Artificial signal test

In this section, we illustrate the abilities of EMD and its variations such as EEMD, CEEMDAN, and impCEEMDAN and the reconstruction method. As an example, we propose a mode mixing example with a low-frequency oscillation and a linear trend. A sustained pure tone plus a gapped one with a higher frequency will inevitably show us mode mixing when applied to EMD due to the local nature of the method. The analyzed signal is $signal = c_1 + c_2 + c_3 + c_4$ with

$$c_1 = \begin{cases} 0 & \text{if } 1 \leq n \leq 500 \\ \sin(2\pi 0.255(n - 501)) & \text{if } 501 \leq n \leq 750 \\ 0 & \text{if } 751 \leq n \leq 1000 \end{cases} \quad (14)$$

$$c_2 = \sin(2\pi 0.065(n - 1)) \quad (15)$$

$$c_3 = \sin(2\pi 0.005(n - 1)) \quad (16)$$

$$c_4 = 0.002(n) \quad (17)$$

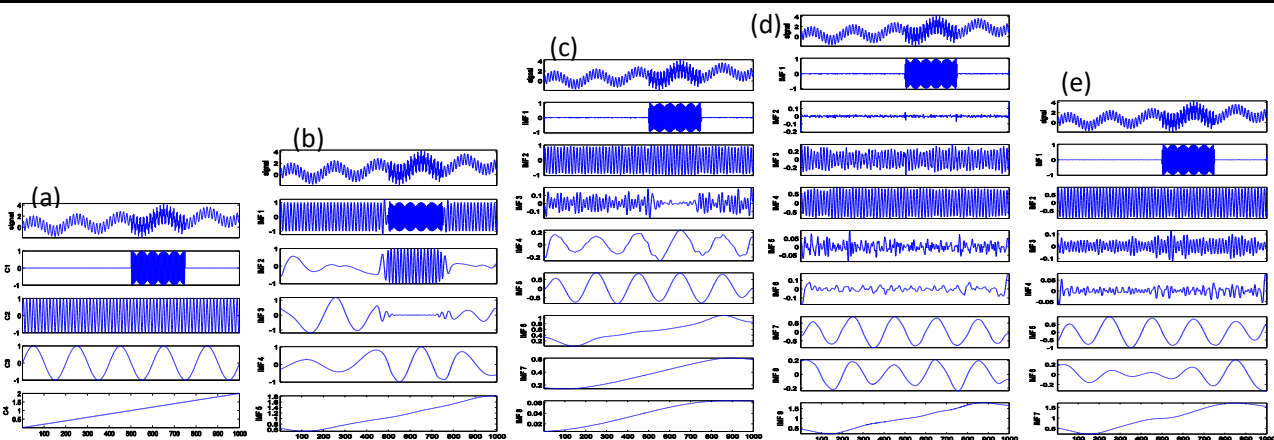


Fig. 1 (a) Test signal and its components, and resulting IMFs decomposed by (b) EMD, (c) EEMD, (d) CEEMDAN and (e) impCEEMDAN

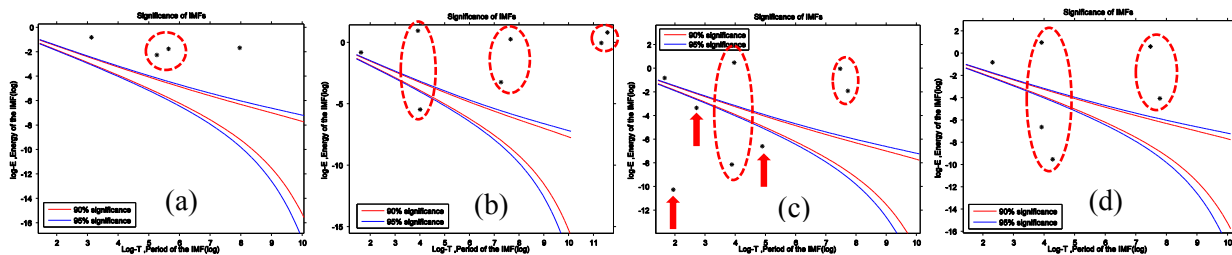


Fig. 2 Results of statistical significance tests of IMFs obtained from (a) EMD, (b) EEMD, (c) CEEMDAN and (d) impCEEMDAN. The upper bound (90th and 95th percentiles) and lower bound (5th and 10th percentiles) are determined from the probability distribution of the energy density of the IMFs from Gaussian white noise. The asterisks from left to right correspond to the pairs of the normalized energy density and the mean period of IMF1 to the last IMF for each case. When the IMF energy is located above the upper bound or below the lower bound, then that IMF should be considered to contain information at that selected confidence interval. IMFs within the red circle can be combined to form a single IMF based on the reconstruction requirements.

The c_1 , c_2 , c_3 , and c_4 can be considered to represent sea-level changes due to an intermittent signal such as waves and storm surge, tides, seasonal variations, and long-term trend, respectively (see Fig. 1).

RESULTS

Typical decomposition results for the four methods are presented in Fig. 1. The mode mixing problem is clearly exhibited in the first four IMFs from EMD in Fig. 1(b). For the three noise-assisted methods the higher frequency intermittent signal is well recovered. However, every ensemble of test signal plus noise was completely decomposed independently from each other and then a total number of eight, nine, and seven modes are obtained, although from the third mode onwards they have very small energy level compared to the original test signal. In the resulting IMFs from EEMD, the energy levels of the last two residual trends are very low (Fig. 1(c)). The last two trends, IMF7 and 8 in Fig. 1(c), are actually spurious modes generated due to the different residual noise at each realization (ensemble). This symptom is already noticed in Chambers (2015) such that the added noise cannot be filtered completely, then propagates into low frequency modes, and finally generates spurious modes in low frequency band. Therefore, when applying the EEMD method for decomposition, care is needed in terms of the number of ensemble simulations and the number of resulting IMFs. A spurious second mode appears in original CEEMDAN modes in Fig. 1(d) due to residual noise with similar scale of the signal. It also produces the low-energy modes such as the fifth and sixth derived from the fourth of the CEEMDAN result. The result of impCEEMDAN shows better performance with no mode mixing, no spurious low frequency modes, and no spurious second mode (Fig. 1(e)). However, the c_2 , c_3 , and c_4 are not reproduced perfectly. We have also tested the sensitivity of the number of realization (ensemble) in EEMD and found that as the number of realizations increases, the probability of having different number of modes increases, and so does the reconstruction error (Lee, 2016).

After decomposing the time series, the statistical significant test was carried out. Figure 2 illustrates the energy level of each IMF with asterisks for IMFs from high-frequency in left to low-frequency in right. Based on the reconstruction method described,

a low energy IMF can be combined with a high energy IMF if its energy-density level is below the low bound of the energy-density spread function of the white noise as indicated with red arrows in Fig. 2(c). If the mean period of IMF is not approximately two times shorter or longer than the neighboring IMF, then it can be also combined with the nearest IMF as indicated within the red circle in Fig. 2.

In Fig. 2(a), the second and third IMFs from EMD can be combined because they do not fulfill the first requirement for IMF reconstruction. In the significance test result for EEMD in Fig. 2(b), the third from the left shows very low energy level with a close period with the second one. Therefore, the third and the second IMFs can be combined. The fourth IMF can also be combined with the fifth one, and the sixth and the seventh can be combined too. In the significance test result for CEEMDAN in Fig. 2(c), the fourth and fifth IMFs can be combined to form a single IMF, and the seventh and the eighth IMFs can be combined to form a single IMF, based on the reconstruction conditions. The third and sixth IMFs are statistically insignificant but their average periods satisfy the requirement. However, we combined IMFs from the third to the sixth for comparison of reconstruction results. Lastly, in the significant test of impCEEMDAN in Fig. 2(d), the IMFs from the second to the fourth can be combined to form a single IMF, and the fifth and the sixth IMFs can be combined to form a new IMF3.

Based on the reconstruction results, the new set of IMFs from EMD, EEMD, CEEMDAN and impCEEMDAN are presented in Fig. 3. Overall, the reconstruction method performs very well in reducing insignificant IMFs, but the mode mixing in EMD and the spurious IMFs in low frequency in EEMD and the spurious mode due to residual noise in high frequency in CEEMDAN are still remained. On the other hand, the reconstructed result from impCEEMDAN exhibits very good performance in IMF1 and IMF2. The new IMF3 shows also very good agreement with the c_3 signal. The results of EEMD and CEEMDAN still produce larger number of IMFs than the test signal components, while the EMD and impCEEMDAN result in the same number of IMFs with the test components.

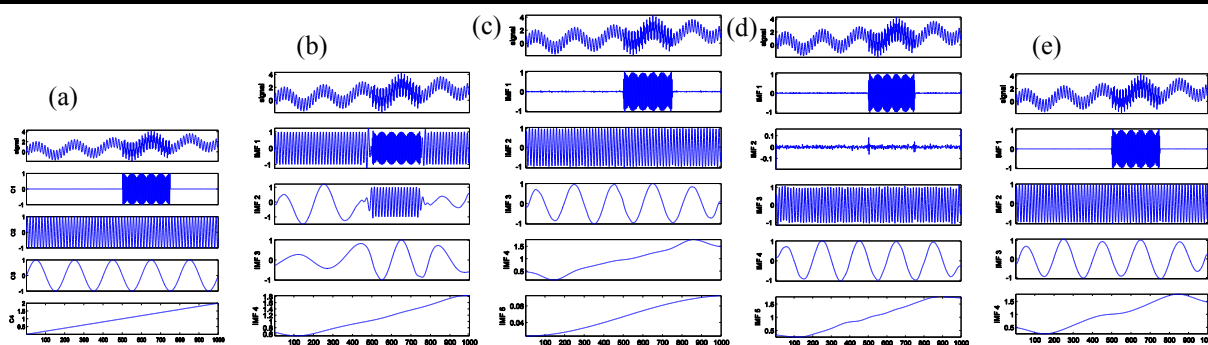


Fig. 3 (a) Test signal and its components, and reconstructed IMFs from (b) EMD, (c) EEMD, (d) CEEMDAN and (e) impCEEMDAN based on the results of statistical significant test

CONCLUSIONS

Empirical mode decomposition (EMD) and its variations such as EEMD, CEEMDAN and impCEEMDAN are introduced and investigated for their capability in terms of mode separations in the case of sea level records. Artificial signal tests illustrate that the residual noise and spurious low-frequency in EEMD and high-frequency in CEEMDAN modes are improved in impCEEMDAN decomposition. Moreover, the reconstruction method for decomposed IMFs is proposed based on the relationship between energy-density level of IMFs and energy-density spread function of Gaussian white noise from statistical significance test. The artificial signal test shows a good reconstructed result that the component signals are well reproduced particularly in impCEEMDAN method.

The EMD and its variations are useful tools for non-linear and non-stationary data analysis. However, those issues still remained to be solved such as mode mixing in EMD, spurious low frequency modes in EEMD and a spurious high-frequency mode in CEEMDAN. Therefore, it is of critical importance that an extra care is needed when applying them to sea-level records and interpreting the resulting low frequency modes. Finally, in this study, the *end effect* in EMD and its variations is found and has to be improved in further study.

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