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Predictive Formulas for Breaker Depth Index and Breaker Type

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ABSTRACT

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This study was undertaken to establish the most reliable breaker depth index formula that will yield satisfactory predictions for a wide range of hydraulic and beach conditions. The applicability of six existing formulas for computing the breaker depth index was examined with the use of a large amount of published laboratory data (524 cases collected from 22 sources). The behavior of these formulas was studied with respect to the main governing parameters, which are the beach slope and the deep-water wave steepness. It was found that most formulas show quite good predictions for cases including gentle slopes ($0.01 < m \leq 0.07$). However, the predictions are typically not satisfactory for breaking waves on steep slopes ($m > 0.1$), and the formulas do not always present a physically correct behavior with respect to the two main parameters. A new formula is proposed to predict the breaker depth index with the best possible accuracy. Also, a discussion is included on the application of the formula to random waves and on the relationship between the breaker depth index and the Irribaren number to distinguish between different breaker types.

ADDITIONAL INDEX WORDS: *Breaking wave, breaker depth index, breaker type, random waves, beach slope, deep-water wave steepness.*

INTRODUCTION

To analyze or simulate coastal processes (*e.g.,* to estimate surf zone hydrodynamics in studies on nearshore morphology), the breaking wave height is one of the essential quantities to determine because it characterizes the most dynamic phenomenon in the nearshore zone. Thus, to compute wave height transformation in the surf zone, it is necessary to determine the initiation of breaking and the breaker type. A spilling breaker creates intense turbulence mainly at the surface, whereas a plunging breaker generates a large vortex that penetrates below the water surface. (TING and KIRBY, 1995; WANG, 1998; WANG and KRAUS, 1999). GALVIN (1968) observed a relationship between the breaker type and the Irribaren number ξ_∞, defined as,

$$\xi_\infty = \frac{m}{\sqrt{\lambda_\infty}} \tag{1}$$

where m is the slope of the beach and $\lambda_\infty = H_\infty/L_\infty$ is the deep-water (offshore) wave steepness (H_∞ and L_∞ are, respectively, the deep-water wave height and wavelength).

A simple way to estimate the location of the breaker line is to compute the wave height-to-water depth ratio where the wave will break (breaker depth index γ_b), which seems, according to different authors (GALVIN, 1972; GODA, 1970; MICHE, 1944; WEGGEL, 1972), to be a function mainly of the offshore wave steepness and the mean beach slope. RATTANAPITIKON and SHIBAYAMA (2000) compared many existing formulas and showed that they differed significantly and induced large uncertainties when slopes deviated from the typical range (*i.e.,* $m < 0.02$ or $m > 0.1$). Because these formulas are semiempirical and often based on a limited data set, their accuracy over large data ranges might not be so good.

In this paper, we compare six existing formulas for calculating γ_b, with an extensive data set. On the basis of this comparison, a new semiempirical formula is proposed involving the beach slope and the offshore wave steepness that displays improved behavior and agreement with the data. Laboratory data on breaking wave height employed in this paper have been compiled from various sources, as summarized in Table 1. Also, because the Irribaren number is often used to distinguish between different breaker types, a discussion is included on the relationship between the breaker depth index and the Irribaren number.

BREAKER DEPTH INDEX

Introduction

The breaker depth index is commonly used to define the wave height at breaking (HORIKAWA, 1988, pp. 79–88; VAN RIJN, 1990, pp. 298–305).

$$\gamma_b = \frac{H_b}{h_b} \tag{2}$$

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Table 1. *Summary of compiled laboratory data on breaker depth index and breaker types.*

Source	No. of Experiments	Beach Conditions	m	λ_∞
Iversen (1952)	63	Plane beach	0.033–0.2	0.003–0.080
Horikawa and Kuo (1966)	60	Step beach	0.00	0.013–0.080
Horikawa and Kuo (1966)	98	Plane beach	0.01–0.05	0.006–0.073
Galvin (1968)	4 (47)*	Plane beach	0.05–0.20	0.001–0.051
Galvin (1969)	19 (19)*	Plane beach	0.05–0.20	0.001–0.051
Saeki and Sasaki (1973)	2	Plane beach	0.02	0.005–0.039
Iwagaki *et al.* (1974)	23 (23)*	Plane beach	0.01–0.05	0.005–0.074
Walker (1974)	15	Plane beach	0.03	0.001–0.037
Van Dorn (1978)	12	Plane beach	0.022–0.083	0.001–0.03
Singamsetti and Wind (1980)	95	Plane beach	0.025–0.2	0.018–0.079
Mizuguchi (1980)	1	Plane beach	0.01	0.045
Nadaoka and Kondoh (1982)	14 (14)*	Plane beach	0.03	0.013–0.080
Visser (1982)	7 (7)*	Plane beach	0.05–0.10	0.014–0.079
Maruyama *et al.* (1983)	1	Plane beach	0.03	0.091
Stive and Battjes (1984)	2	Plane beach	0.03	0.01–0.032
Battjes and Stive (1985)	20	Plane beach	0.025–0.07	0.01–0.03
Okayasu, Shibayama, and Nimura (1986)	(10)*	Plane beach	0.033–0.05	0.01–0.09
Stive and Wind (1986)	(2)*	Plane beach	0.025	0.028–0.032
Okazaki and Sunamura (1989)	(134)*	Plane beach	0.05–2	0.001–0.117
Smith and Kraus (1990)	5 (5)*	Plane beach	0.03–0.044	0.008–0.096
Smith and Kraus (1990)	75 (75)*	Barred beach	0.033	0.009–0.092
Christensen and Deigaard (2001)	3 (3)*	Plane beach	0.05–0.074	0.015–0.025
Ting (2002)	5 (5)*	Plane beach	0.0286	0.015–0.04

* Data for breaker type available.

where h is the water depth and subscript b denotes the point of incipient breaking.

A second parameter, the breaker height index $\Omega_b = H_b/H_\infty$ is also widely used (SMITH and KRAUS, 1990). However, according to the present authors, it induces a greater uncertainty in the prediction of H_b. Indeed, for deep-water wave steepnesses up to 0.02, this parameter is less sensitive than Equation (2) because predictions by existing formulas are approximately between 1 and 1.1 (*cf.* RATTANAPITIKON, VIVATTANASIRISAK, and SHIBAYAMA, 2003; SMITH and KRAUS, 1990).

Many different formulas for γ_b have been proposed on the basis of monochromatic wave experiments. RATTANAPITIKON and SHIBAYAMA (2000) and SMITH and KRAUS (1990) compared some of these formulas with data. It appears that the main governing factors for γ_b are, as for the breaker type, the bottom slope m and the deep-water steepness λ_∞. According to the results of RATTANAPITIKON and SHIBAYAMA (2000), six formulas are interesting to compare.

WEGGEL (1972)

$$\gamma_b = b(m) - a(m)\frac{H_b}{L_\infty}$$

$$a(m) = 43.75[1 - \exp(-19m)]$$

$$b(m) = \frac{1.56}{1 - \exp(-19.5m)} \tag{3}$$

BATTJES (1974)

$$\gamma_b = 1.062 + 0.137 \log(\xi_\infty) \tag{4}$$

OSTENDORF and MADSEN (1979)

$$\gamma_b = 0.14\frac{L_b}{h_b}\tanh\left\{[0.8 + 5\min(m, 0.1)]\frac{2\pi h_b}{L_b}\right\} \tag{5}$$

SINGAMSETTI and WIND (1980)

$$\gamma_b = 0.937m^{0.155}\lambda_\infty^{-0.13} \tag{6}$$

SMITH and KRAUS (1990)

$$\gamma_b = \frac{1.12}{1 + \exp(-60m)} - 5.0[1 + \exp(-43m)]\lambda_\infty \tag{7}$$

GODA (1970) modified by RATTANAPITIKON and Shibayama (2000)

$$\gamma_b = 0.17\frac{L_\infty}{h_b}\left\{1 + \exp\left[\frac{\pi h_b}{L_\infty}(16.21m^2 - 7.07m - 1.55)\right]\right\} \tag{8}$$

It should be noted that Equations (3), (5), and (8) require an iterative technique, whereas Equations (4), (6), and (7) allow for direct computation. The range of validity for these formulas is implicitly assumed to be the range of the data used for their calibration. Both the formulas by SINGAMSETTI and WIND (1980) and SMITH and KRAUS (1990) were calibrated with their own data set only (*cf.* Table 1). The formulas by BATTJES (1974), OSTENDORF and MADSEN (1979), and WEGGEL (1972) are mainly based on the GALVIN (1968, 1969) and IVERSEN (1952) data set and thus have a similar range of validity ($0.02 < m \leq 0.2$). Finally, the GODA (1970) formula modified by RATTANAPITIKON and SHIBAYAMA (2000) was calibrated with a similar data set to that compiled for this paper (*i.e.,* $0.00 < m \leq 0.5$).

Comparison with Compiled Data Set

Table 2 summarizes the errors in the predictions of γ_b for the six studied formulas and for the newly developed formula

Table 2. *Summary of results (predictive errors) obtained with the different formulas.*

Formula	P20 Plane	P20 Step	P20 Bar	P20 All	E_{rms} Plane	E_{rms} Step	E_{rms} Bar	E_{rms} All
Weggel (1972)	66.5	81.5	64.0	67.0	4.3	2.1	6.5	4.3
Ostendorf and Madsen (1979)	81.0	70.0	76.0	79.0	1.9	2.6	2.5	2.1
Singamsetti and Wind (1980)	73.5	0	62.5	64.0	3.5	100	6.0	14.7
Battjes (1974)	83.5	0	65.5	71.5	2.0	100	4.5	13.4
Smith and Kraus (1990)	81.0	23.5	56.0	71.0	2.1	7.7	3.6	2.9
Goda (1970) modified	79.5	78.5	77.5	79.0	2.3	1.9	2.4	2.3
Present study	84.5	80.0	85.5	84.0	1.9	1.9	1.9	1.9

Formula	P10 All	All data ($\Delta\gamma_b$) Mean	All data ($\Delta\gamma_b$) SD	All Data ($\Delta\gamma_b$), $m > 0$ Mean	All Data ($\Delta\gamma_b$), $m > 0$ SD
Weggel (1972)	40.0	+0.097	0.155	+0.108	0.144
Ostendorf and Madsen (1979)	45.5	−0.037	0.305	−0.030	0.138
Singamsetti and Wind (1980)	39.5	−0.025	0.144	+0.070	0.154
Battjes (1974)	44.5	−0.081	0.281	+0.006	0.140
Smith and Kraus (1990)	44.5	−0.050	0.154	−0.029	0.146
Goda (1970) modified	48.0	+0.013	0.133	+0.015	0.135
Present study	51.5	−0.015	0.131	−0.011	0.132

discussed later, including error estimates for all the data as well as for the cases with a plane-sloping beach, a step beach, and a barred beach. The quantities P10 and P20 denote the percentage of values obtained with an error of less than 10% and 20%, respectively, whereas E_{rms} is the root mean square relative error (%), defined as

$$E_{rms} = 100\sqrt{\sum \frac{(\gamma_{b,pred} - \gamma_{b,exp})^2}{\gamma_{b,exp}^2}} \qquad (9)$$

where $\gamma_{b,pred}$ is the predicted breaker depth index and $\gamma_{b,exp}$ is the breaker depth index observed experimentally. The results obtained for a step beach (Horikawa and Kuo, 1966) and for a barred beach (Smith and Kraus, 1990) were separated out because these cases present a different situation than the classical plane-sloping beach. Experiments on a step beach ($m = 0$) induce significant reflection, which can influence the wave breaking process. In the same way, Smith and Kraus (1990) observed that the bar, depending on its shape, can induce a strong seaward return flow, which could influence wave breaking. Thus, error estimates are presented for all data, as well as separately for the data with regard to the plane-sloping beach, the step beach, and the barred beach.

The mean value and the standard deviation (SD) of the difference $\Delta_b = \gamma_{b,pred} - \gamma_{b,exp}$ is also included (computed for all the data and the data for which $m > 0$). The figures in Appendix A show the comparison between predicted and observed breaker depth index obtained for the six formulas with the use of all the experimental data.

The best results for the existing formulas and the use of all the data are obtained with the Battjes (1974), Goda (1970), and the modified Ostendorf and Madsen (1979) formulas. The latter formula also yields the best results for the barred beach data. Finally, the Weggel (1972) formula shows the best results for the step beach experiments, although it gives a constant value $\gamma_b = 0.78$ independent of the deep-water wave conditions. Some formulas can reach an agreement with the data of close to 80% for a permitted error of 20%, whereas none reaches 50% for a permitted error of 10%. This deviation can partly be explained by the errors that come from the experimental measurements since the breaker height is typically visually estimated. Also, the break point can be defined in several ways. Singamsetti and Wind (1980) listed seven possible definitions, which could induce additional errors. Closer inspection of the graphs (see Appendix A) allows for the following comments.

- Weggel's (1972) formula generally overestimates the breaker depth index and produces considerable dispersion of the results. This overestimation could occur because Weggel intended his formula to yield an envelope to the observed data to obtain a high estimate on the breaker height appropriate for design.
- Ostendorf and Madsen's (1979) formula gives good agreement with the data but seems to exhibit a discrete behavior (same computed value for different experimental data), and no value can be computed for a beach with a slope equal to zero ($\gamma_b = 0$, if $m = 0$).
- Singamsetti and Wind's (1980) formula generally overestimates the breaker depth index and produces considerable dispersion of the results.
- Battjes' (1974) formula yields overall good results, but no value can be computed for a beach with a slope equal to zero ($\gamma_b = 0$, if $m = 0$).
- Smith and Kraus' (1990) formula often underestimates the breaker depth index.
- The modified Goda (1970) formula gives less dispersion in the results but tends to overestimate for small values on the breaker depth index and to underestimate for large values.

Dependence of Breaker Depth Index on Deep-Water Wave Steepness and Beach Slope

With regard to the dependence of γ_b on the deep-water wave steepness, the behavior of each of the studied formulas is not entirely satisfactory. The graphs in Appendix B.1 present the results of comparing data and formulas for varying λ_∞ under fixed bed slope. Although it is quite difficult to observe any trend in the experimental data of Horikawa and Kuo (1966) ($m = 0$, $m = 0.0125$; see graphs in Appendix B.1a and 1b), because significant dispersion exists in the data, a decrease in γ_b with an increase in λ_∞ remains the principal tendency in the other data sets. This decrease becomes clearer and quite pronounced in the data from Iwagaki *et al.* (1974), Singamsetti and Wind (1980), and Smith and Kraus (1990) for mild slopes (see Appendix B.1c–f). Only the Smith and Kraus (1990) formula displays a correct behavior in accordance with this observation. The five other formulas seem insensitive for high values on λ_∞. For larger values of m (see Appendix B.1g), this sensitivity is not so apparent, mainly because of the lack of data. Finally, for smaller values of λ_∞, most of the data indicate a value of γ_b that is nearly independent of λ_∞, with a decrease in λ_∞ (see Appendix B.1c–e with the data of Galvin [1968, 1969], Iversen [1952], Van

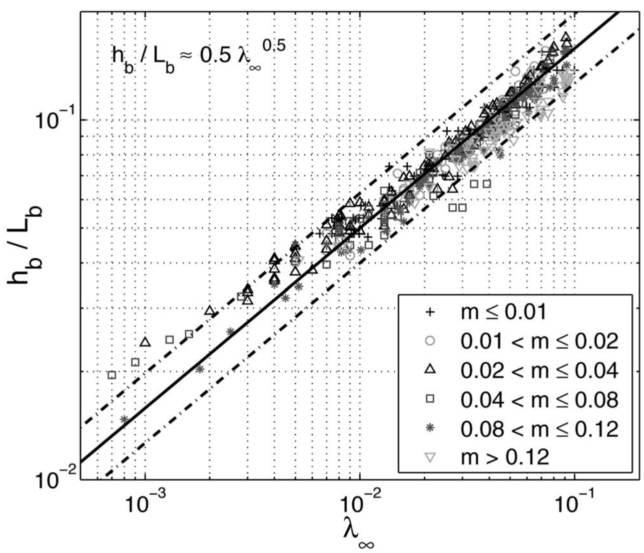

Figure 1. The ratio h_b/L_b as a function of the deep-water wave steepness λ_∞ using all the data available (the straight line corresponds to Equation [12] with $A_b = 0.5$, and the dashed lines to a prediction within a factor of 1.25).

DORN [1978] and WALKER [1974]). Both the BATTJES (1974) and the SINGAMSETTI and WIND (1980) formulas show unsatisfactory predictions for those specific cases. To summarize the influence of the deep-water wave steepness on the breaker depth index, the best observed behavior is displayed by the SMITH and KRAUS (1990) formula for mild slopes and by the modified GODA (1970) formula for slopes up to 10%.

The beach slope, m, has often been the only parameter used for calibration of semiempirical formulas, mainly on the basis of MICHE's (1944) results. Thus, good agreement is observed for most of the formulas for mild slopes (*i.e.,* γ_b displays an exponential dependence on m; see graphs in Appendix B.2). According to the data (except for the step beach experiment), the SMITH and KRAUS (1990) formula yields the best results for slopes below 0.1. However, for larger slopes, the tendency changes, and γ_b becomes a decreasing function of m (see graphs B.2 with Smith and Kraus' data) after having reached a maximum for $m \approx 0.2$. Above this value, the BATTJES (1974) and the SINGAMSETTI and WIND (1980) formulas display incorrect behavior. The OSTENDORF and MADSEN (1979), SMITH and KRAUS (1990), and WEGGEL (1972) formulas, which all assume a fixed value for $m > 0.1$, give better agreement with data, but it is not satisfactory. Only the GODA (1970) formula modified by RATTANAPITIKON and SHIBAYAMA (2000) shows a correct behavior for beach slopes up to 0.2. For lower values on the deep-water wave steepness ($\lambda_\infty < 0.03$), it seems like the maximum γ_b is reached for a milder slope ($m \approx 0.1$), and the observed decrease with larger slopes is smoother.

A New Formula For Breaker Depth Index

One aim of this study was to propose a new formula for γ_b that exhibits the best possible behavior with respect to the dependence on the mean slope m and the offshore wave steepness γ_∞. The new formula is based on MICHE's (1944) results similar to the OSTENDORF and MADSEN (1979) formula, but it employs the offshore wave steepness instead of the ratio h_b/L_b to allow for a direct computation of the breaker depth index.

The MICHE (1944) formula can be written as

$$\gamma_b = 0.142 \frac{L_b}{h_b} \tanh\left[f(m,\ \lambda_\infty)\frac{2\pi h_b}{L_b}\right] \qquad (10)$$

where L_b is the wavelength at the break point. The function $f(m,\ \lambda_\infty)$ depends on the bottom slope and was empirically introduced by OSTENDORF and MADSEN (1979) with $f(m,\ \lambda_\infty) = 0.8 + 5 \min(m,\ 0.1)$ ($f(m,\ \lambda_\infty) = 1$ in case of the MICHE [1944] formula).

Assuming shoaling follows the first-order Stokes theory, the wavelength can be expressed as a function of its deep-water value $L_\infty = gT_w^2/(2\pi)$ and the water depth h,

$$L_w = L_\infty \tanh kh \approx \sqrt{2\pi h L_\infty} \quad \text{if } kh \ll 1 \qquad (11)$$

(if $kh \ll 1$, $k = \sqrt{2\pi/[hL_\infty]}$). Because $h_b = \alpha_b H_\infty/\gamma_b$, where $\alpha_b > 1$ is the shoaling coefficient at the break point.

$$\frac{h_b}{L_b} = \sqrt{\frac{h_b}{2\pi L_\infty}} = A_b\sqrt{\lambda_\infty} \qquad (12)$$

where $A_b = \sqrt{\alpha_b/(2\pi\gamma_b)}$ is assumed to be approximately constant. It appears from Figure 1 that $A_b \approx 0.5$ is a reasonable value over a wide range of steepness and slope values.

With Equation (12), Equation (10) can be rewritten in an explicit form.

$$\gamma_b = \frac{0.284}{\sqrt{\lambda_\infty}} \tanh[f_*(m,\ \lambda_\infty)\pi\sqrt{\lambda_\infty}] \qquad (13)$$

Following the method proposed by RATTANAPITIKON and SHIBAYAMA (2000), an empirical function can be fit to the data to obtain the function $f_*(m,\ \lambda_\infty)$

$$f_*(m,\ \lambda_\infty) = \frac{1}{\pi\sqrt{\lambda_\infty}} \operatorname{arctanh}\left(\frac{\gamma_b\sqrt{\lambda_\infty}}{0.284}\right) \qquad (14)$$

Table 3 presents a comparison between the results from the iterative method (Equation [10]) and the explicit method

Table 3. *Comparison between the iterative method (Equation [10]) and the explicit method (Equation [13]) with the use of the original* f (m, λ_∞) *or the fit* f_* (m, λ_∞).

Authors	Original f_* (= f)	P20 (%)	E_{rms}	Fit f_*	P20 (%)	E_{rms}
Miche (1944)	1	98	2.2	—	—	—
Ostendorf and Madsen (1979)	$0.80 + 5.0m$	95	2.6	$0.89 + 4.5m$	95	3.2
Rattanapitikon and Shibayama (2000)	$0.91 + 5.0m - 11m^2$	94	3.0	$0.88 + 6.1m - 13m^2$	95	3.1

(Equation [13]) with a special fitting of $f_*(m, \lambda_\infty)$, where P20 is the percentage of data points in agreement within a factor of 1.2. If $f_* = 1$, Equation (13) yields a formula quite close to MICHE's (1944) formula. Assuming that $f(m, \lambda_\infty) = f(m)$, OSTENDORF and MADSEN (1979) fit a straight line using data in the range of $0.1 < m < 0.1$. RATTANAPITIKON and SHIBAYAMA (2000) fit a polynomial function of m using a similar data set as the one compiled in this study. The fourth column presents a fit with the use of the present data set and Equation (13). Similar results are found for the formulas. The differences appear to be mainly due to the different data sets used by the authors for the fitting (P20 and E_{rms} not improved). It appears that employing Equation (13) does not reduce the predictive skill of the formula compared with the iterative relationships. More than 95% of the predicted data points are in agreement within a factor of 1.2. Some differences occur for the small values of γ_b predicted by the iterative formula, which explains the apparent dispersion of the results ($E_{rms} > 2$).

These expressions for f (and f_*) are only dependent on the mean slope, although a clear influence from the deep-water wave steepness can be observed in the graphs (Appendix C). Moreover, the polynomial function proposed by RATTANAPITIKON and SHIBAYAMA (2000) diverges rapidly for large slopes. Instead, a sinusoidal function of the beach slope is introduced.

$$f_*(m, \lambda_\infty) = A_1 + A_2 \sin\left[\frac{\pi}{2}\left(\frac{m}{m_{max}}\right)^{\alpha}\right] \qquad (15)$$

where m_{max} is the beach slope for which f_* reaches its maximum value, and A_1, A_2, and α are fitting parameters that are functions of λ_∞.

The graphs in Appendix C present f_* as a function of m together with the experimental data for three different intervals of λ_∞. The best polynomial fit (second order) and the final relationship obtained by the authors are included. A maximum of f_* is clearly observed and seems to be a linear function of λ_∞. The following relationship is proposed.

$$m_{max} = 0.10 + 1.6\lambda_\infty \qquad (16)$$

In the same way, the coefficient α, which represents the "curvature" of the sinusoidal function (for $m \le m_{max}$ and $m > m_{max}$), can be related to the deep-water wave steepness according to the formulas in Equation (17).

$$\alpha = 1 + 14\lambda_\infty \quad \text{if } m \le m_{max}$$

$$\alpha = -(1 + 20\lambda_\infty) \quad \text{if } m > m_{max} \qquad (17)$$

Finally, the best fit is obtained (Equations [18] and [19]).

$$A_1 = 0.87 \qquad (18)$$

$$A_2 = 0.32 + 14\lambda_\infty \qquad (19)$$

Figure 2 presents the result when plotting predictions with the proposed formula against all data, showing an improvement compared with the previous relationships.

Errors obtained with the new formula are summarized in Table 2. It is interesting to see that only 15% of the results for all beach types are found to have an error greater than 20%, and less than 50% of the results for an error greater than 10%. Predictions for plane, barred, and step beaches are the best among the studied formulas, but, as explained in the previous section, experimental results for step beaches (experiment of HORIKAWA and KUO, 1966) can imply an overestimation of γ_b because of possible reflection. However, it can be noted that (as for the BATTJES [1974], GODA [1970], OSTENFORD and MADSEN [1979], and SMITH and KRAUS [1990] formulas) the maximum predicted value by Equation (15) is 1.15, although some observed values reach 1.6. It is a result of the formula being fit to a large amount of scattered data. Finally, because of the errors and uncertainties in the experimental data, it seems that the quadratic error of less than 1.9% is difficult to achieve. One of the great improvements with the new formula is that a correct behavior with regard to beach slope and deep-water wave steepness is obtained for most of the cases (unlike previous formulas).

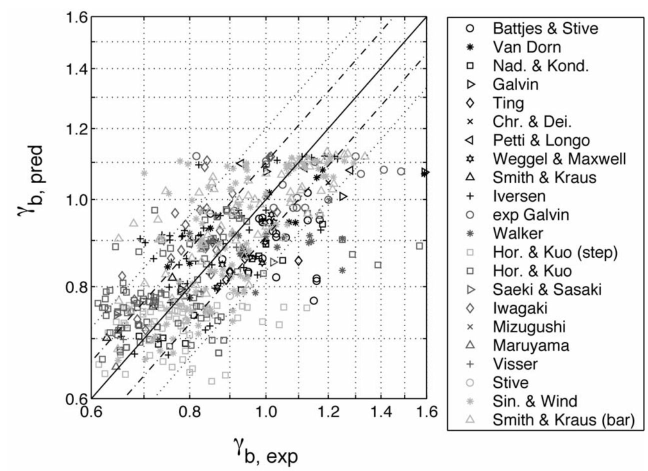

Figure 2. Comparison between experimental data and the proposed formula (Equation [15]).

The distribution of the difference $\Delta(\gamma_b) = \gamma_{b,pred} - \gamma_{b,exp}$ (*cf.* Figure 3) follows a Gaussian probability density function (pdf), which implies that errors correspond mainly to the experimental uncertainties or randomness in the breaking process itself. Because Equations (13) and (15) yield a prediction of the mean value of the observed experimental data, they cannot predict extreme observed values. Looking at the figures in appendix B, it can be observed that, for similar conditions (same slope and offshore wave steepness), a large scatter appears depending on the experimental data. Thus, in an application of Equations (13) and (15), a random variation might be employed to achieve more realistic predictions of γ_b.

Application to Random Wave Model

Modeling random waves in the surface zone follow basically two different approaches:

1. assume a pdf, valid everywhere, and use this function in the wave transformation calculations to obtain the statistical wave properties;

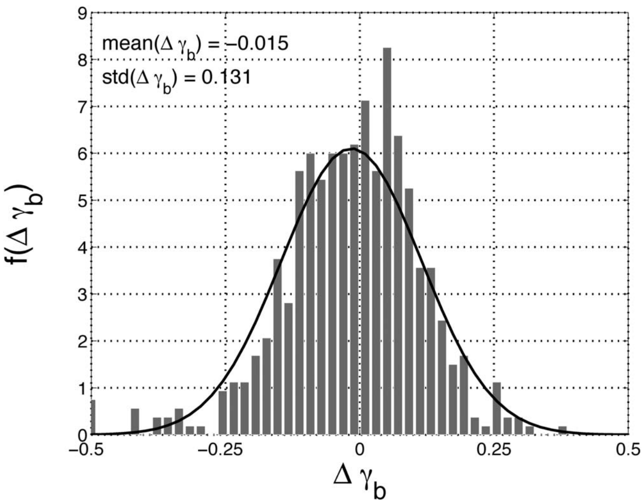

Figure 3. Distribution of the difference $\Delta_{\gamma b} = \gamma_{b,pred} - \gamma_{b,exp}$ with the new formula (the black line corresponds to the Gaussian function).

2. assume a pdf in the offshore, represented by a number of individual wave components, compute the evolution of each monochromatic wave, and determine the local pdf by adding together the different components.

The calculation of the wave transformation is determined by applying an energy flux balance. For shore-parallel depth contours, it reads,

$$\frac{d(EC_g)}{dx} = -D \tag{20}$$

where E is the wave energy, C_g the group velocity, and D the time-averaged energy dissipation due to broken waves and frictional losses as the bed (x-axis points onshore). Assuming that the friction at the bottom is negligible and that the energy dissipation at each wave front corresponds to that in a bore or a hydraulic jump, the energy dissipation can be written as

$$D = \frac{\rho g h H^3}{T(4h^2 - H^2)} Q_b \tag{21}$$

where H and T are the wave height and period, respectively, and Q_b is the proportion of broken waves.

In case of approach 1 described above, $H = H_{rms}$ (root mean square wave height) and $T = T_p$ (peak wave period). Many of the random wave models are based on the approach of BATTJES and JANSSEN (1978). The wave heights in the surf zone are characterized by a Rayleigh distribution, which is assumed to be valid for both broken and unbroken waves. The usual criterion employed to truncate the distribution is that a wave is breaking when its height H_i exceeds some fraction of the water depth. So, to a fixed water depth h, there corresponds a critical wave height H_{ib} over which all the waves are broken (see Figure 4). The quantity Q_b is found by integrating the Rayleigh distribution over all waves for which $H_i > H_{ib}$ (Equation [22]).

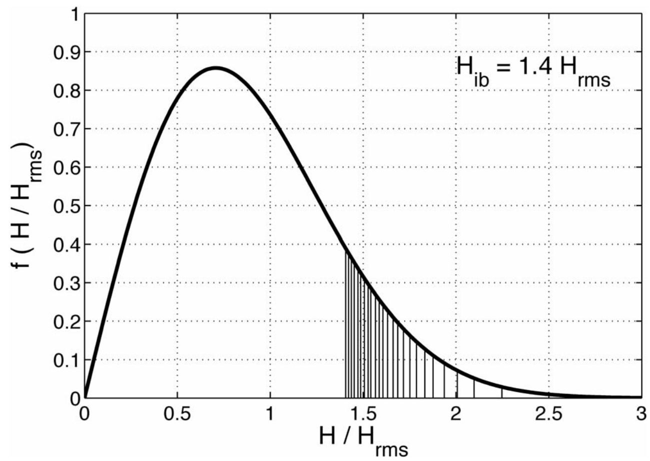

Figure 4. Theoretical wave height distribution and fraction of breaking waves (breaking waves correspond to the shaded area).

$$Q_b = \exp\left[-\left(\frac{H_{ib}}{H_{rms}}\right)^2\right] \tag{22}$$

The height H_{ib} is generally evaluated from calibration of the random wave model with data. A common expression developed by BATTJES and STIVE (1985) is based on the MICHE (1944) formula.

$$H_{ib} = \frac{0.88}{k} \tanh\left(\frac{\gamma}{0.88} kh\right) \tag{23}$$

where k is the wave number and γ the breaker parameter. As a first approximation, $\gamma = 0.88$. NAIRN (1990) modified the value proposed by BATTJES and STIVE (1985).

$$\gamma = 0.39 + 0.56 \tanh(33\lambda_\infty) \tag{24}$$

More recently, RUESSINK, WALSTRA, and SOUTHGATE (2003) found a linear relationship with the product of the local wave number and the water depth.

$$\gamma = 0.76kh + 0.29 \tag{25}$$

As RUESSINK, WALSTRA, and SOUTHGATE (2003) discussed, these relationships include not only the physics of the breakers but also encapsulate structural errors in the model, which might explain the different behavior of these calibration parameters compared with "measured" parameters. However, because the definition of H_{ib} corresponds exactly to the breaker depth index for a monochromatic wave, Equations (13) and (15) can be used (*i.e.,* $H_{ib} = \gamma_b h$), which gives acceptable results.

In the case of the second approach $H = H_i$ (individual wave height from the pdf) and a breaker depth index formula is directly applicable because waves are modeled individually. So, $Q_b = 0$ if the individual wave is not broken $H_i/h \leq \gamma_b$ and $Q_b = 1$ if the individual wave is broken.

An example is given in Figure 5 of the calculated evolution of random waves according to both approaches, and two different offshore wave conditions and beach slopes are shown. For the first approach, H_{ib} was estimated by Equations (13)

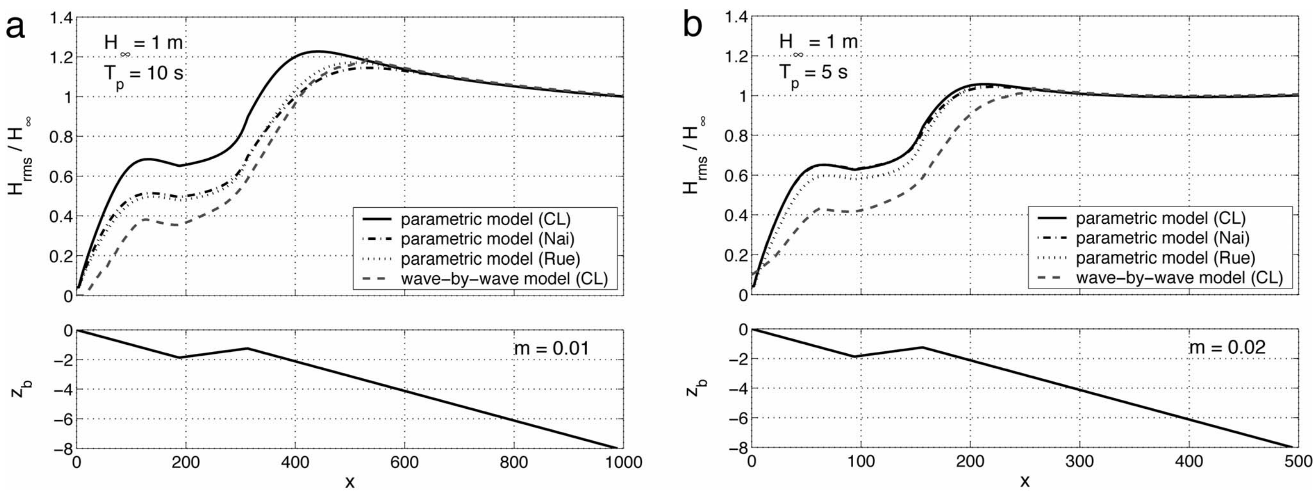

Figure 5. Comparison between the two random wave models with the use of different breaker depth index formulas (CL, Camenen and Larson, Equation [15]; Nai, Nairn [1990]; Rue, Ruessink, Walstra, and Southgate [2003]) for a test case involving a barred beach with two different mean slopes and offshore wave steepnesses. a) A spilling breaking case. b) A plunging breaking case.

and (15) or Equation (23) with Equations (24) or (25). For the second approach, only Equations (13) and (15) were used.

In Figure 5, $\xi_\infty = 0.125$ for both cases; however in Figure 5a, λ_∞ is one-fourth of that in Figure 5b, and the beach slope m is half. In Figure 5a, it appears that the wave-by-wave approach of Equation (15) gives similar results to the parametric approach with a calibrated coefficient. On the other hand, in Figure 5b, the parametric model gives similar results independently of the formula used to calculate H_{ib}. The wave-by-wave approach yields a smaller wave height inside the surf zone. Comparing the wave-by-wave and parametric approaches with Equation (15), it appears that the wave height in the surf zone is always larger with the parametric method. If a coefficient 0.7 ($\sim 1/\sqrt{2}$) is added for the calculation of H_{ib} with Equation (15) similar results are obtained for all conditions. Therefore, assuming that the wave-by-wave approach produces results closer to reality, Equation (15) can be used in a parametric model after introducing a coefficient of 0.7.

BREAKER DEPTH INDEX AND BREAKER TYPE

Breaker Type

Assuming that an accurate estimation of the breaker depth index can be made for all cases, it is interesting to see how this index is related to the breaker type. GALVIN (1968, 1972) defined the following breaker types:

- spilling breakers occur if the wave crest becomes unstable and flows down the front face of the wave producing a foamy water surface;
- plunging breakers occur if the crest curls over the front face and falls into the base of the wave, resulting in a high splash;
- collapsing breakers occur if the crest remains unbroken while the lower part of front face steepens and then falls, producing an irregular turbulent water surface;
- surging breakers occur if the crest remains unbroken and the front face of the wave advances up the beach with minor breaking.

Breaker type is controlled by the bottom slope m and the deep-water steepness λ_∞. BATTJES (1974) used the Irribaren number (or surf similarity parameter) to describe breaker type on the basis of previous results of GALVIN (1968, 1972):

$$\begin{array}{ll} \text{spilling} & \text{if } \xi_\infty \leq 0.5 \\ \text{plunging} & \text{if } 0.5 < \xi_\infty \leq 3.3 \\ \text{surging or collapsing} & \text{if } \xi_\infty > 3.3 \end{array} \qquad (26)$$

Relationship Between Breaker Depth Index and Irribaren Number

Figure 6 shows how measured γ_b varies with ξ_∞. It is clearly seen that γ_b is an increasing function of ξ_∞. The relationship proposed by BATTJES (1974) is a good approximation on the basis of comparison with all the data. However, marked scatter exists, which is the weakness of this formula, as was observed in the previous section.

In Figure 7, a sensitivity study of the new formula (Equation [15]) is made for γ_b against ξ_∞ employing various beach slopes m (Figure 7a) or various deep-water wave steepnesses λ_∞ (Figure 7b). Curves are limited by imposed constraints, *i.e.,* the deep-water wave steepness cannot be greater than 0.14 and the maximum beach slope is fixed as the maximum value from the experimental data ($m \leq 0.5$).

The most interesting result to emerge from these graphs is that they show two different types of behavior of the breaker depth index depending on the breaker type. Indeed, for $\xi_\infty < \xi_{\infty,cr} \approx 0.5$ (spilling waves, cr is critical value), γ_b is mainly sensitive to the offshore wave steepness λ_∞, and for a fixed ξ_∞, γ_b is nearly independent of the slope m. On the contrary, for $\xi_\infty < \xi_{\infty,cr}$ (plunging and surging waves), γ_b is mainly sen-

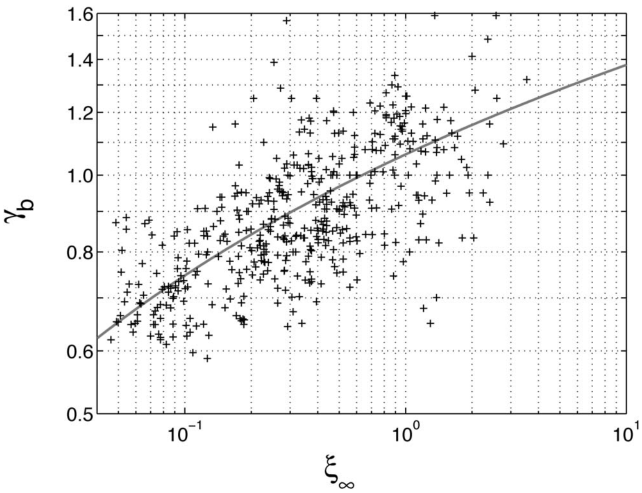

Figure 6. Comparison between experimental data on breaker depth index and the Irribaren number (relationship from Battjes [1974] is also shown, see Equation [4]).

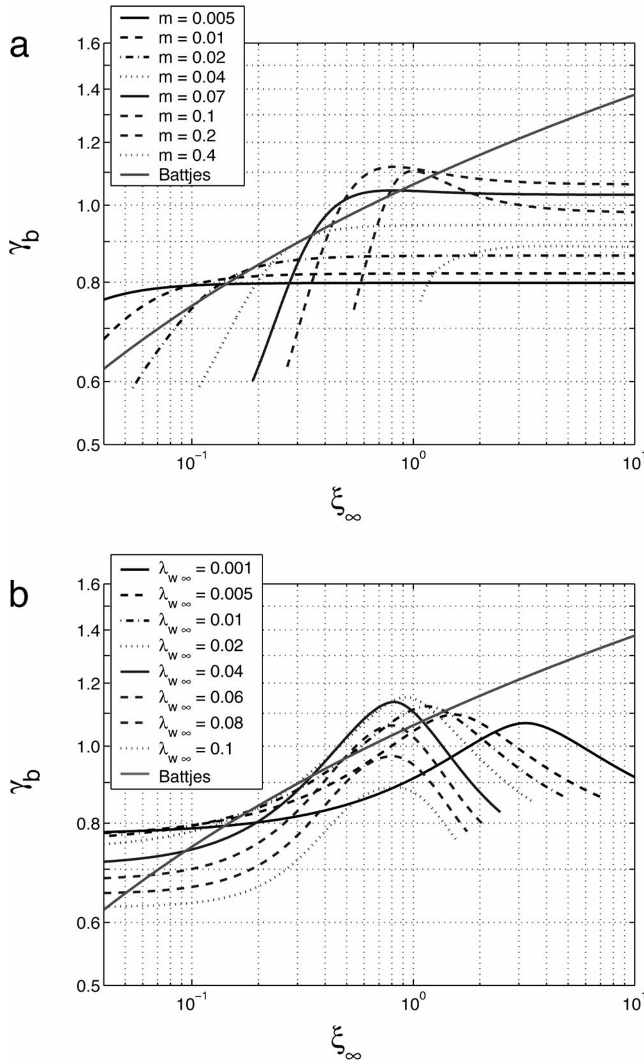

Figure 7. Comparison between the proposed formula (Equation [15]) and the Irribaren number for fixed beach slope (a) and fixed deep-water wave steepness (b).

sitive to the beach slope *m,* and for a fixed ξ_∞, γ_b is nearly independent of λ_∞. Moreover, the maximum values on the breaker depth index (see Figure 7b) seem to correspond to plunging breakers (according to values of λ_∞, this maximum appears for $0.8 < \xi_\infty < 4$). Thus, surging and collapsing breakers only occur for conditions corresponding to the decreasing part of the computed curves. The physical mechanisms controlling these types of breakers do not allow high values of γ_b, even if they were observed for lower values of λ_∞ ($\approx$0.01) and steep beaches ($m > 0.2$). In the same way, a separation point between spilling and plunging waves could more or less be the inflection point of the increasing part of the computed curves (see Figure 7b), which explains the critical ξ_∞ from Galvin (1968, 1972) varying between 0.2 and 0.5, which Smith and Kraus (1990) also noticed.

Figure 8 presents the final result concerning breaker type prediction on the basis of the Irribaren number and the breaker depth index with the use of data in which complete information is available. The data from Okazaki and Sunamura (1989) were also added because they performed an interesting study on step beaches. Unfortunately, the experimental results for the breaker depth index were not presented in their article. Thus, γ_b has been estimated by Equation (15). The symbols used for this data set have been made smaller.

The lower curve (separating of the nonbreaking wave region) corresponds to Equation (15) with $\lambda_\infty = 0.14$. The upper curve (separating of the unstable wave region) corresponds to the maximum values obtained with this equation with a factor of 1.25. The factor 1.25 was employed because the new equation does not compute the maximum values observed from the experimental data. Finally, the three other curves are empirically determined to separate the spilling breaker, plunging breaker, collapsing breaker, and surging breaker regions following the ideas presented in the previous paragraph. Large uncertainties still exist, but they seem to be mainly due to the uncertainties in the available experimental data. This graph provides an improvement for breaker type prediction compared with Galvin's (1968, 1972) relationships (Equations [26]). The separation lines in Figure 8 are defined by the formulas in Equations (27).

spilling/plunging

$$\text{when } \xi_\infty = 1.5 - 1.25\gamma_b \qquad (\xi_\infty = 0.2\text{–}0.6) \tag{27a}$$

plunging/collapsing

$$\text{when } \xi_\infty = 4.4 - 2.5\gamma_b \qquad (\xi_\infty = 0.8\text{–}2.9) \tag{27b}$$

collapsing/surging

$$\text{when } \xi_\infty = 1.7 + 3.3\gamma_b \qquad (\xi_\infty = 3.6\text{–}6) \tag{27c}$$

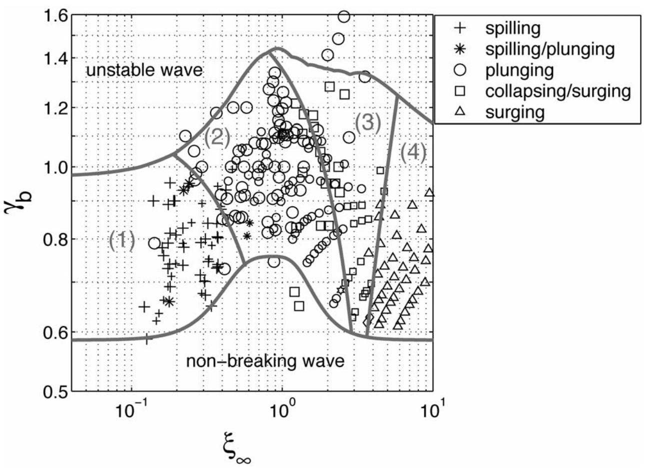

Figure 8. Observation of different breaker types with respect to the Irribaren number and the breaker depth index (1: spilling breaker area, 2: plunging breaker area, 3: collapsing breaker area, 4: surging area).

CONCLUSION

More than 500 experimental cases from 22 published sources covering a wide range of beach slopes and wave conditions were used to compare predictions from six existing breaker depth index formulas. A sensitivity study against beach slope and offshore wave steepness allowed for exploring the limits of these different formulas. The modified GODA (1970) formula, adding a bottom slope effect proposed by RATTANAPITIKON and SHIBAYAMA (2000), improves the behavior of the original formula substantially from slopes up to 0.1 but gives an overestimation for smaller slopes. On the basis of the sensitivity study, a new breaker depth index formula (Equation [13]) was proposed starting with MICHE's (1944) expression and a correction factor that is a function of both beach slope and offshore wave steepness (Equation [15]).

This relationship presents better results when compared with all the data, but above all, it shows an improved behavior with respect to the two studied parameters. Some questions still remain for step beaches because reflection has a great influence. A tentative method was proposed for the application of the new breaker depth index formula in random wave models. With the use of a wave-by-wave model, the formula can be used as it stands (because it corresponds to the summation of the effects of a number of monochromatic waves), whereas a coefficient of 0.7 should be employed with a parametric model. Finally, with this new formula, a more accurate prediction of breaker type (compared with the GALVIN [1968, 1972] equations) was obtained employing the Irribaren number and the breaker depth index.

NOTATION

The following symbols and subscripts are used in this paper.

Variables

A_1, A_2 = empirical coefficients
C = wave speed
C_g = group speed
D = wave energy dissipation
E = wave energy
h = water depth
H = wave height
H_{ib} = critical wave height for the truncation of the wave probability distribution
L = wavelength
m = beach slope
Q_b = proportion of broken waves
T, T_p = wave period and peak wave period
x = horizontal coordinate perpendicular to the beach
α = empirical coefficient
γ_b = breaker depth index
λ = wave steepness
Ω_b = breaker height index
ξ = Irribaren number

Subscripts

b = denotes the point of incipient breaking
∞ = denotes a deep-water value
cr = denotes a critical value
exp = denotes an experimental result
max = denotes a maximum value
pred = denotes a predicted result
rms = denotes a root mean square value

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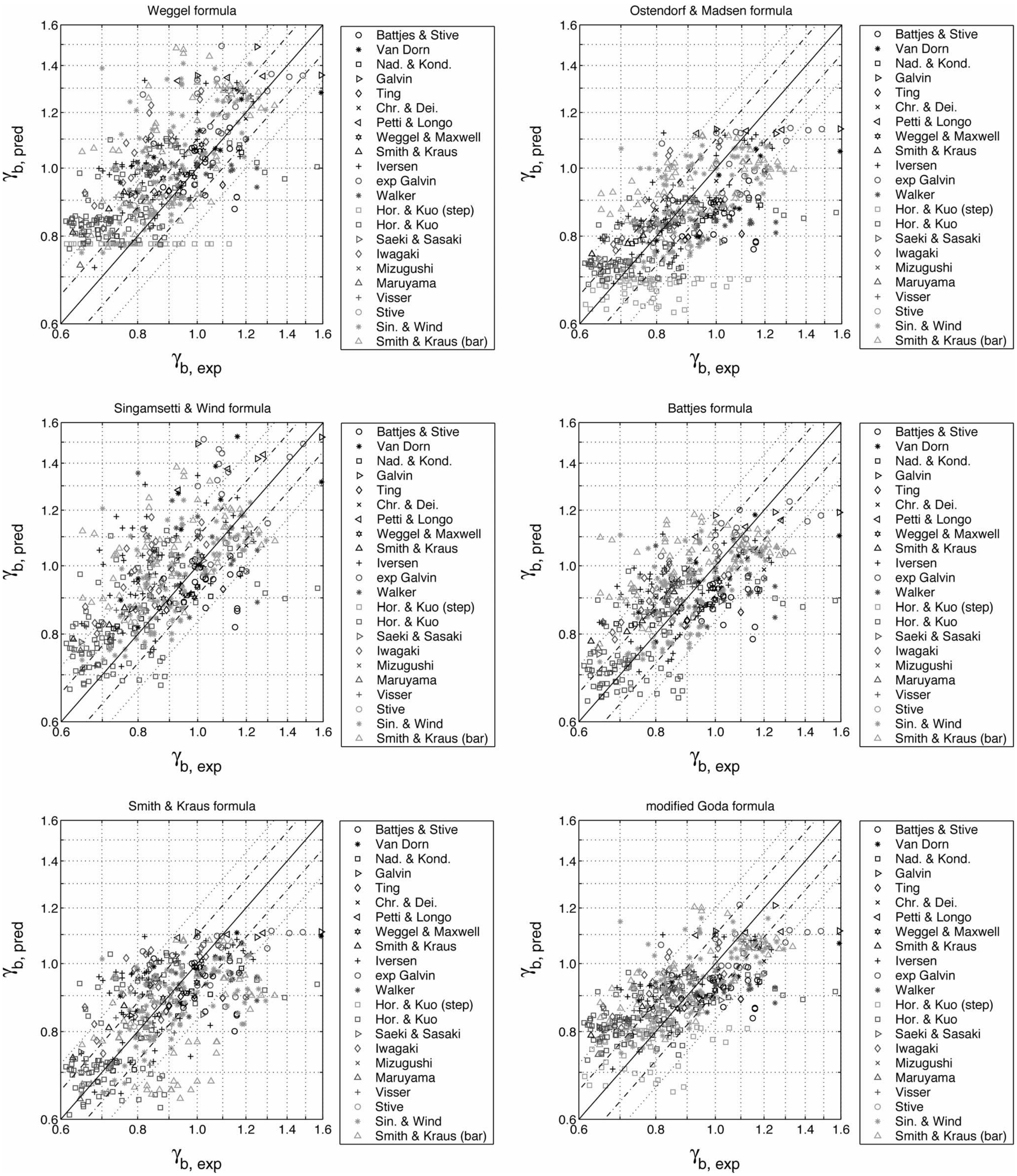
APPENDIX A

Comparison between Experimental Data and the Six Studied Formulas

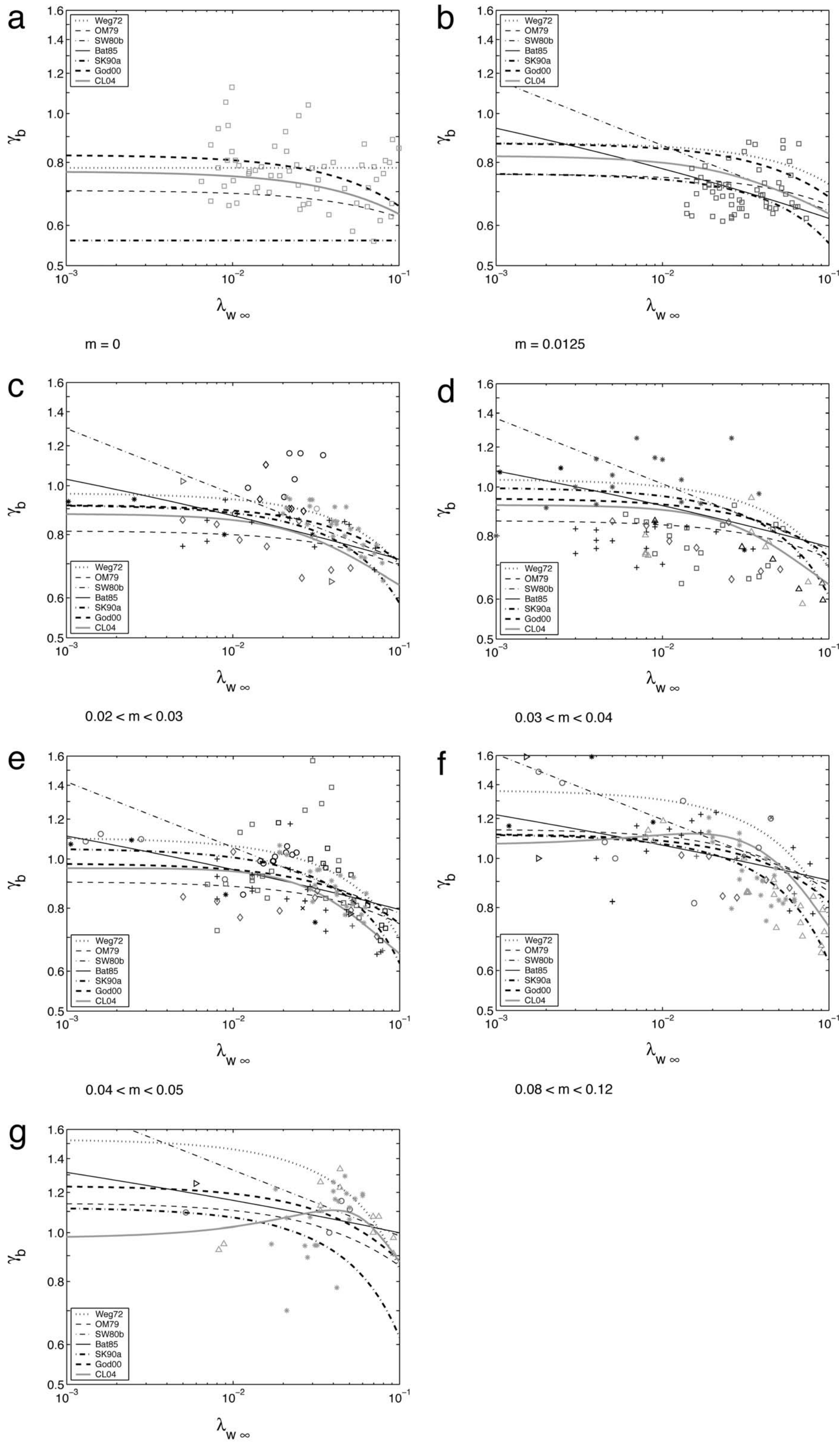


APPENDIX B

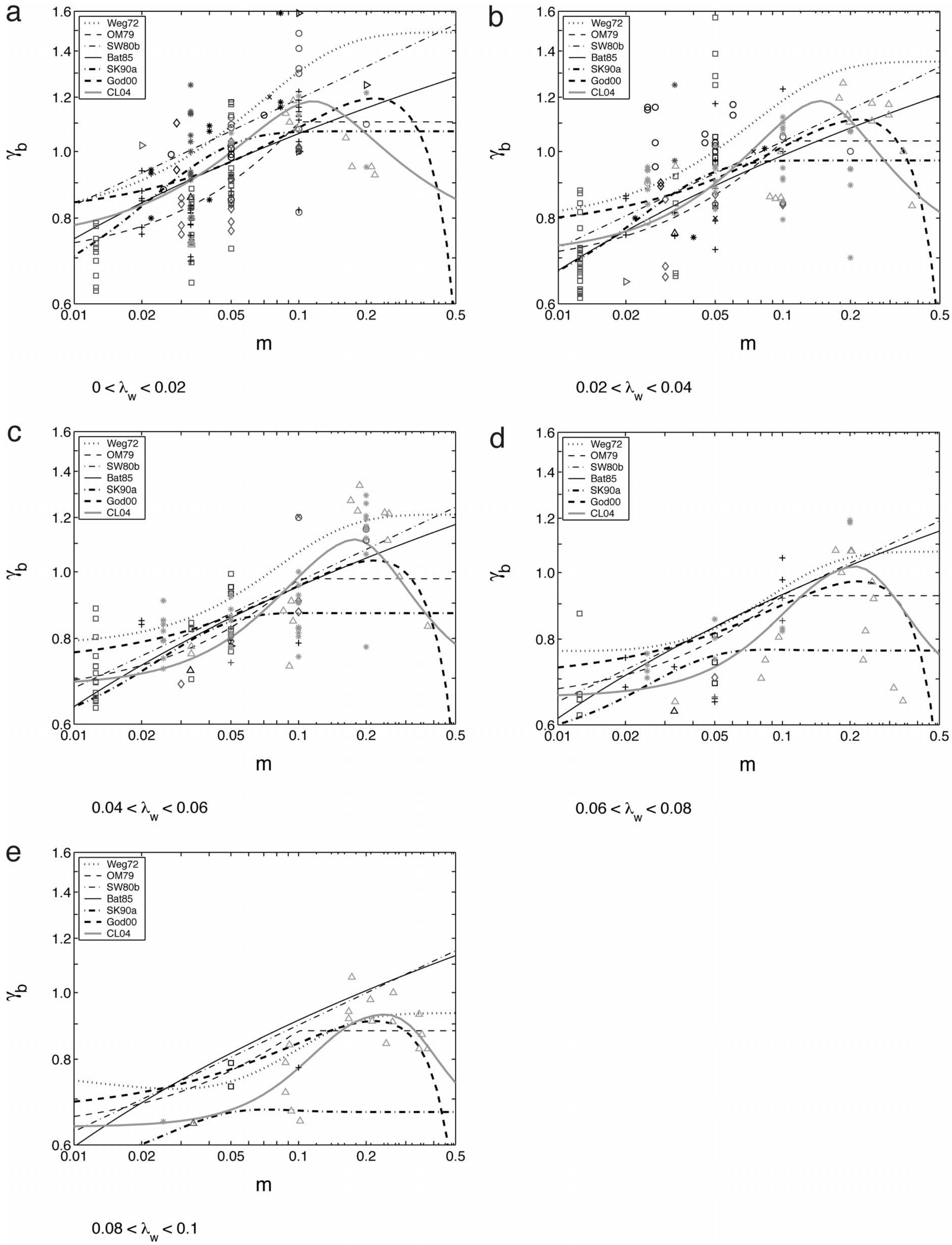
Breaker Depth Index as a Function of Beach Slope or Deep-Water Wave Steepness

In this Appendix, the data on the breaker depth index are displayed as a function of the beach slope (deep-water wave steepness fixed) or as a function of the deep-water wave steepness (beach slope fixed). The curves correspond to the six studied formulas plus the new one (symbols for the experimental data correspond to those used in the plots of Appendix A).

B.1. Breaker Depth Index as a Function of Deep-Water Wave Steepness



B.2. Breaker Depth Index as a Function of the Beach Slope



APPENDIX C

Fit of the Function $f_*(m, \lambda_\infty)$: Influence of Beach Slope m (a, c, and e) and Wave Steepness λ_∞ (b, d, and f)