Evaluation of the Penman-Monteith (FAO 56 PM) Method for Calculating Reference Evapotranspiration Using Limited Data

Authors: Mario Córdova, Galo Carrillo-Rojas, Patricio Crespo, Bradford Wilcox, and Rolando Célleri

Source: Mountain Research and Development, 35(3) : 230-239
Published By: International Mountain Society
URL: https://doi.org/10.1659/MRD-JOURNAL-D-14-0024.1
Evaluation of the Penman-Monteith (FAO 56 PM) Method for Calculating Reference
Evapotranspiration Using Limited Data

Application to the Wet Páramo of Southern Ecuador

Mario Córdova1, Galo Carrillo-Rojas2,3, Patricio Crespo1, Bradford Wilcox4, and Rolando Célérier1

1 Departamento de Recursos Hídricos y Ciencias Ambientales & Facultad de Ciencias Agropecuarias, Universidad de Cuenca, Víctor Manuel Albornoz y los Cerezos, Campus Balzay, Cuenca, Ecuador
2 Departamento de Recursos Hídricos y Ciencias Ambientales & Facultad de Ciencias Químicas, Universidad de Cuenca, Víctor Manuel Albornoz y los Cerezos, Campus Balzay, Cuenca, Ecuador
3 Laboratory for Climatology and Remote Sensing, Faculty of Geography, University of Marburg, Deutschhausstraße 10, D-35032 Marburg, Germany
4 Department of Ecosystem Science and Management, Texas A&M University, College Station, Texas, USA

Open access article: please credit the authors and the full source.

For the first step, calculating $ET_o$, the United Nations Food and Agriculture Organization (FAO) adopted the Penman-Monteith method in its Irrigation and Drainage Paper No. 56. Known as FAO 56 PM, this method is a global standard based on meteorological data (Allen et al 1998), and it has been found to work well in numerous locations if the required data are available (Allen et al 1989; García et al 2004; López-Urrea et al 2006; Xing et al 2008). The FAO 56 PM method requires measurements of temperature, relative humidity, wind speed, and solar radiation. This data demand is the main constraint on its use in locations where climate data are limited (Stöckle et al 2004; Trajkovic and Kolakovic 2009a; Li et al 2012; Rahimikhoob et al 2012). This is a common problem in developing countries (Droogers and Allen 2002; Exner-Kittridge and Rains 2010; Gocic and Trajkovic 2010; Tabari 2010; Hou et al 2013) and especially for tropical regions (Wohl et al 2012) and high-altitude areas (Kollas et al 2014), as highlighted in Figure 1.
To overcome this difficulty, the FAO 56 PM method includes procedures for estimating meteorological data using other, more commonly measured variables such as minimum and maximum temperature (Allen et al. 1998). These procedures have been tested in a variety of conditions, including a semiarid location in Tunisia (Jabloun and Sahli 2008), a cold humid location in Canada (Sentelhas et al. 2010), and 2 temperate locations—one in Bulgaria (Popova et al. 2006) and the other in Korea (Kwon and Choi 2011). But they have been tested only to a very limited extent for high-elevation landscapes. Another method, the Hargreaves method, which allows estimation of $E_{To}$ on the basis of temperature data only, has been tried in a few studies, focused on the Bolivian altiplano (Garcia et al. 2004), Florida (Martinez et al. 2010), Iran (Fooladmand and Haghighat 2007), Tanzania (Igbadun et al. 2006), China (Xu et al. 2013), and in 1 case, global patterns (Droogers and Allen 2002).

Climate data are often limited for mountain environments, even though they occupy close to 25% of the continental surface (Beniston 2006), are home to a quarter of the global population (Meybeck et al. 2011), and directly or indirectly provide sustenance and water for more than half of the global population (Beniston 2006). In the tropics, 1 mountain region of particular importance is the high-elevation grassland of the northern Andes known as the páramo. The páramo is the major source of water for the Andean highlands of Venezuela, Colombia, and Ecuador, much of the adjacent lowland areas, and the arid coastal plains of northern Peru (Buytaert, Célleri et al. 2006). In Ecuador, the Andean highlands provide water for hydropower, agriculture (De Bièvre et al. 2003), and domestic and industrial uses, as well as numerous environmental services (Célleri and Feyen 2009). Like all mountainous regions, the páramo is topographically complex, which not only gives rise to dramatic differences in climate over short horizontal and vertical distances (Becker and Bugmann 1997), but also presents a particularly challenging environment for meteorological monitoring.

Reliable estimates of $E_{To}$ are needed for many hydrological, ecological, and agricultural applications. In light of the paucity of climate data for tropical high mountains, our study of the Ecuadorian páramo aimed to evaluate the accuracy of the FAO 56 PM method for calculating $E_{To}$ using different combinations of incomplete climate data. We tested 6 cases, each with a different type and/or number of missing variables.

Materials and methods

Meteorological data measurements

The meteorological data for this study came from 2 automatic weather stations, both located in the high-elevation páramo of Ecuador: 1 in the Zhurucay river basin (79.24°W; 3.06°S; 3780 masl) on the Pacific side of the Andes, from which we obtained 2 years of data (March 2011–February 2013), and 1 near Toreadora Lake (79.22°W; 2.78°S; 3979 masl) on the Atlantic side, from which we obtained 1 year of data (2013) (Figure 2). At each site, temperature, relative humidity, wind speed, and solar radiation were recorded every 5 minutes. Both stations have excellent quality data, in accordance with the standards outlined in Allen (1996). We also obtained temperature data from a conventional weather station of the Instituto Nacional de Meteorología e Hidrología (National Institute of Meteorology and Hydrology) installed in the vicinity. By comparing our data with the long-term data from this station (1963 to present), we confirmed that the years considered in our study did not present anomalies.
To avoid bias due to differences in sensor accuracy and precision, both weather stations were equipped with the same sensor configuration. Air temperature and relative humidity were measured by means of a Campbell Scientific CS-215 combined probe with radiation shield. A Met-One 034B Windset anemometer was used to measure wind speed. Finally, solar radiation was measured using a CS300 Apogee pyranometer manufactured by Campbell Scientific. Table 1 shows average values for the meteorological variables measured at the study sites, as well as maximum and minimum values for temperature.

**ET₀ calculations**

Daily $ET₀$ was calculated for both stations by means of the FAO 56 PM equation (Allen et al 1998):

$$ET₀ = \frac{0.408\Delta (R_a - G) + \gamma \frac{900}{T + 273} u_2 (e_s - e_a)}{\Delta + \gamma (1 + 0.34 u_2)},$$

where $ET₀$ is the reference evapotranspiration (mm day$^{-1}$); $R_a$ is the net radiation at the crop surface (MJ m$^{-2}$ day$^{-1}$), which was estimated according to the procedures outlined by Allen et al (1998); $G$ is the soil heat-flux density (MJ m$^{-2}$)}
day\(^{-1}\)), which can be assumed as zero for daily calculations according to Allen et al (1998); \(T\) is the mean daily air temperature (\(^\circ\)C) at a height of 2 m; \(u_2\) is the wind speed at a height of 2 m (m s\(^{-1}\)); \(e_s\) is the saturation vapor pressure (kPa); \(e_a\) is the actual vapor pressure (kPa), which is based on relative humidity measurements; \((e_s - e_a)\) is the saturation vapor pressure deficit (VPD) (kPa); \(\Lambda\) is the slope of the vapor pressure curve (kPa \(^{\circ}\)C\(^{-1}\)); and \(\gamma\) is the psychrometric constant (kPa \(^{\circ}\)C\(^{-1}\)).

As noted earlier, the use of the FAO 56 PM equation requires a complete data set. For situations in which climate data are incomplete, the authors (Allen et al 1998) proposed an alternative method that substitutes estimated values for the missing meteorological variables. These estimates are determined as described below.

**Solar radiation:** Solar radiation \((R_s)\) is estimated as a function of minimum and maximum air temperature, on the assumption that differences between maximum and minimum temperature are governed by the daily \(R_s\) at a given location, as proposed by Hargreaves and Samani (1985):

\[
R_i = k_{R_s} \sqrt{(T_{\text{max}} - T_{\text{min}})} R_s,
\]

where \(R_s\) is the extraterrestrial radiation (MJ m\(^{-2}\) day\(^{-1}\)), \(T_{\text{max}}\) is the maximum air temperature (\(^\circ\)C), \(T_{\text{min}}\) is the minimum air temperature (\(^\circ\)C), and \(k_{R_s}\) is the adjustment coefficient (\(^{\circ}\)C\(^{-0.5}\)). For this study we used \(k_{R_s} = 0.16\) because the study area is in an interior region where air masses are not influenced by large water bodies (Allen et al 1998).

**Relative humidity:** When relative humidity (RH) data are lacking, the actual vapor pressure \((e_a)\) can be calculated by assuming that dewpoint temperature \((T_{\text{dew}})\) is close to \(T_{\text{min}}\). This is usually the case at sunrise at reference weather stations (Allen et al 1998). This assumption appears to reflect the typical conditions of the p\'aramo. RH reaches saturation 66\% and 82\% of the days in Toreadora and Zhurucay, respectively, and it reaches values higher than 95\% almost every day (94\% and 97\% of the days for Toreadora and Zhurucay, respectively). Thus, we assumed that \(RH = 100\%\) when \(T_{\text{min}}\) occurs, and on that basis we calculated \(e_a\) in kPa as follows:

\[
e_a = e'(T_{\text{min}}) = 0.611 \times e^{\left(\frac{17.6T_{\text{min}}}{T_{\text{min}} - 237}\right)}
\]

where \(e'(T_{\text{min}})\) is the function described on the right side, \(T_{\text{min}}\) is the minimum temperature, and \(e\) is the exponential function.

**Wind speed:** When no wind data are available, Allen et al (1998) proposed using average wind speeds measured in a nearby location within the same homogeneous region. But because the scarcity of meteorological monitoring in the Andean p\'aramo made this option impossible, we instead used a second option suggested by Allen et al (1998): We assumed \(u_2 = 2\) m s\(^{-1}\) (an average value from 2000 stations around the world).

**Estimations of \(E_{To}\) with different combinations of missing data:** We calculated \(E_{To}\) for 3 cases of single missing variables: wind speed \((-u_2)\), solar radiation \((-R_s)\), and relative humidity \((-RH)\); and for 3 cases of combinations of missing variables: \(-R_s\) and \(-u_2;\) \(-R_s\) and \(-RH;\) and \(-R_s\), \(-RH;\) and \(-u_2). All 6 of these \(E_{To}\) calculations were compared with calculations made on the basis of the complete data set, to assess the accuracy of the FAO 56 method for calculating \(E_{To}\) when data are missing.

In addition, we evaluated the temperature-based Hargreaves method (Equation 4), to compare the results with those obtained by the FAO 56 PM method for estimating \(E_{To}\) when only temperature data were available:

\[
E_{To} = 0.408 \times 0.0023(T_{\text{max}} + 17.8)(T_{\text{max}} - T_{\text{min}})^{0.5} R_s,
\]

where \(T_{\text{max}}\) is the mean of \(T_{\text{max}}\) and \(T_{\text{min}}\), \(R_s\) is extraterrestrial radiation, and the 0.408 coefficient is the conversion factor for MJ m\(^{-2}\) day\(^{-1}\) to mm day\(^{-1}\).

**Data analysis**

In accordance with previous work (eg Jacovides and Kontoyiannis 1995; Garcia et al 2004; Popova et al 2006; Sentelhas et al 2010; Kwon and Choi 2011), we also used mean bias error (MBE) and root mean square error (RMSE) to evaluate the quality of \(E_{To}\) calculations made with incomplete climate data. We computed MBE and RMSE in mm day\(^{-1}\) using Equations 5 and 6, respectively.

In addition, we used percentual mean bias error (\%MBE) (Equation 7) because in the p\'aramo, the low \(E_{To}\) rates yield lower MBE values than those obtained in other regions (eg arid and semiarid climates), which could lead to misinterpretation of the results:

\[
\text{MBE} = \frac{1}{n} \sum_{i=1}^{n} \left( E_{T_{\text{ref}}} - E_{T_{\text{est}}} \right)
\]
where \( E_{\text{Toest}} \) is the reference evapotranspiration calculated with incomplete data, \( E_{\text{Toref}} \) is the reference evapotranspiration calculated with complete data sets, and \( n \) is the number of days.

To evaluate the performance of the FAO 56 PM method for the different cases of missing data, we followed the criteria described in Table 2. It should be noted that the criteria for maximum permissible errors are subjective and depend on the particular application (Annandale et al. 2002).

**Results**

Calculations of \( E_T \) became progressively less accurate as the number of estimated variables increased, with \( E_{\text{To}} \) being overestimated for both locations (Table 3; Figure 3). When only wind data were missing, the calculations were excellent. When only wind and RH data were missing, \( E_{\text{To}} \) was under-estimated. But this figure reveals an interesting dichotomy. During days with low \( E_{\text{To}} \), calculations for all 6 cases of missing variables were better for the months of June through August—even those with the most limited data. However, calculations for the September–May time frame, all of which were missing \( R_s \) data, were much poorer. At the same time, calculations based on data missing only wind speed or RH were better during the entire year. Calculations of \( E_T \) for the Zhurucay site exhibited the same general behavior for all 6 cases of estimated variables, except that the VPD estimations exhibited the same general behavior for all 6 cases of estimated variables, except that the VPD estimations based on data without RH did not perform as well as those for Toreadora (Table 3; Figure 3).

Bar charts showing MBE for the 6 cases of estimated variables are presented in Figure 5. Clearly, calculations are excellent if only wind speed data are missing. But this figure reveals an interesting dichotomy. During days with low \( E_{\text{To}} \), calculations based on each of the different cases overestimated \( E_{\text{To}} \). However, during days when \( E_{\text{To}} \) exceeded 3 mm, the same calculations underestimated \( E_{\text{To}} \). The magnitude of under- or overestimation varied according to which variables were missing, consistent with the patterns discussed previously.

**Table 2** Performance evaluation criteria for the FAO 56 PM method of calculating \( E_T \) using incomplete data.

<table>
<thead>
<tr>
<th>Percentual Mean Bias Error</th>
<th>Quality of the Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;5%</td>
<td>Excellent</td>
</tr>
<tr>
<td>5–10%</td>
<td>Good</td>
</tr>
<tr>
<td>10–15%</td>
<td>Acceptable</td>
</tr>
<tr>
<td>&gt;15%</td>
<td>Poor</td>
</tr>
</tbody>
</table>

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (E_{\text{Toest}} - E_{\text{Toeq}})^2}
\]

\[
\%MBE = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{E_{\text{Toest}} - E_{\text{Toeq}}}{E_{\text{Toeq}}} \right) \times 100,
\]

where \( E_{\text{Toeq}} \) is the reference evapotranspiration calculated with incomplete data, \( E_{\text{Toef}} \) is the reference evapotranspiration calculated with complete data sets, and \( n \) is the number of days.

\( u_2 \) = wind speed at 2 m height; \( RH \) = relative humidity; \( R_s \) = solar radiation; MBE = mean bias error; \%MBE = percentual mean bias error; RMSE = root mean square error.
Discussion

Analyzing previous work on the subject, we found that the results were quite good for a Bulgaria site (Popova et al 2006) but mixed for other locations. In general, a lack of wind speed data was not a major source of error in the humid climates, but it was in the semiarid climate. For RH, a dichotomy was observed: lack of RH data leads to overestimation of $E_{To}$ in humid climates and to underestimation in semiarid climates. Finally, when the model was used to estimate $R_s$ on the basis of maximum and minimum temperatures, it worked poorly for humid conditions but yielded quite good results for semiarid conditions.

Our study is the first to comprehensively evaluate the effects of substituting estimates for missing data in calculations of $E_{To}$ in the Andean wet páramo. We found that if the only missing variable was wind speed data, calculations of $E_{To}$ were excellent. For this case, we used the global average of 2 m s$^{-1}$ (Allen et al 1998), which is very close to the average wind speed at the Toreadora site (2.3 m s$^{-1}$). Interestingly, using the global average wind speed at the Zhurucay site also worked well, even though the actual average at this location was 3.6 m s$^{-1}$. This would suggest that $E_{To}$ calculations may not be very sensitive to wind speed in cold humid climates, such as that of the wet páramo. Sentelhas et al (2010), working in the Great Lakes region of Canada, also obtained good estimates of $E_{To}$ in the absence of wind speed data (they used data from a nearby weather station). In contrast, in the semiarid climate of Tunisia (Jabloun and Sahli 2008), $E_{To}$ predictions made without actual measurements of wind speed were poor. In arid and semiarid climates, wind speed is clearly an important determinant of $E_{To}$ because of the importance of the aerodynamic term under these dry and high wind speed conditions (Garcia et al 2004).

With the FAO 56 method, RH data are required for calculating actual vapor pressure ($e_o$). In the absence of RH information, $e_o$ is estimated on the basis of minimum temperature. The problem with using this method for humid climates is that condensation occurs during the night, which leads to an overestimation of VPD (Allen et al 1998) and a resultant overestimation of $E_{To}$ — as indeed we observed at our sites. Calculations for the Zhurucay site were less accurate than those for the Toreadora site (Table 3), because the higher RH at Zhurucay (Table 1) led to a greater overestimation of VPD. Sentelhas et al (2010) had similar findings at their Canadian sites: In the absence of RH data, $E_{To}$ was overestimated when the FAO 56 method was used. In contrast, in Tunisia (Jabloun and Sahli 2008), $E_{To}$ was underestimated when RH data were missing. This is because in arid climates the air is not saturated at

![Boxplots showing daily $E_{To}$ calculations for the Toreadora and Zhurucay sites, based on each of the 6 cases of incomplete data, compared with calculations based on complete data sets. Each box lies between the 0.25 and 0.75 quartiles, and the central line is the median. The whiskers indicate the range of the data within the minimum and maximum values.](https://bioone.org/journals/Mountain-Research-and-Development)
FIGURE 4  Daily and annual ETₐ at the Toreadora site (10 days running average) for the complete data set and the 6 cases of incomplete data.

FIGURE 5  Mean bias error for different rates of daily ETₐ for the Toreadora and Zhurucay sites. The 6 bars in each group represent the 6 cases of incomplete meteorological data, and each group corresponds to an ETₐ range.
minimum temperature and condensation does not necessarily occur, which leads to underestimation of VPD and hence underestimation of $E_{To}$.

Finally, we found that $R_s$ data are extremely important for calculating $E_{To}$ in the wet páramo of southern Ecuador. This has been observed in Canada as well (Sentelhas et al. 2010), where calculations done without $R_s$ data yielded the maximum error. For both our wet páramo locations, calculations of $E_{To}$ made without $R_s$ yielded high error values; although the result was poor at Toreadora, it was acceptable at Zhurucay (Table 3). Because of lower cloud cover, average daily $R_s$ is higher at Zhurucay (13.90 MJ m$^{-2}$) than at Toreadora (12.13 MJ m$^{-2}$). The higher values observed at Zhurucay are closer to the average of estimated values of around 16 MJ m$^{-2}$ day$^{-1}$ obtained at both sites. Likewise, Sentelhas et al. (2010) found that when actual $R_s$ is lower than 20 MJ m$^{-2}$ day$^{-1}$, the FAO method led to systematic overestimation of this variable. These results for cold and humid conditions contrast with those of the study carried out in Tunisia (Jabloun and Sahli 2008), where calculations made without measured $R_s$ data yielded the lowest MBE.

When we evaluated the effects on the $E_{To}$ calculations of the 3 combinations of missing variables ($-R$, $-RH$, and $-R_s$; $-R$, $-RH$, and $-u_2$; $-R$, $-RH$, $-R_s$, and $-u_2$), all the estimations were poor. The least accurate estimates were found when all 3—$RH$, $R_s$, and $u_2$—were missing, as has been shown in previous studies (Popova et al. 2006; Jabloun and Sahli 2008; Trajkovic and Kolakovic 2009b; Sentelhas et al. 2010; Kwon and Choi 2011). Next in line was the combination with both $R_s$ and RH missing. However, when both $R_s$ and $u_2$ were missing, the results were not significantly different from those obtained when only $R_s$ was missing, which strengthens the hypothesis that $E_{To}$ calculations are not very sensitive to wind speed in this climate. These findings are almost identical to those of Sentelhas et al. (2010) in Canada.

When incomplete data are used to calculate $E_{To}$ for high-elevation conditions, the full range of $E_{To}$ is not captured (Figure 3). We believe there are 2 primary reasons for this:

- At these elevations, $E_{To}$ is at its maximum under clear-sky conditions, when $R_s$ is exceptionally high. $R_s$ was underestimated under clear-sky conditions, likely because the method is temperature based and was developed for more temperate and lower-elevation regions.
- The method did not capture the low $E_{To}$ periods that occur under cloudy conditions and very high RH.

Under these conditions, the method’s procedures for estimating data overestimate both $R_s$ and VPD.

We also found that results were better from May to September than for the other months of the year (Figure 4). During these months, the use of estimated values for missing data coincided better with calculations based on the complete data set. May to September is the period for which $R_s$ estimates are the best. Additional information on the annual cycle of $R_s$, its estimation, and why estimates are more accurate during this time of the year is given in Supplemental material, Appendix S1: (http://dx.doi.org/10.1659/MRD-JOURNAL-D-15-0024.S1).

Our evaluation of the Hargreaves method showed that it overestimated $E_{To}$ as it usually does under humid conditions (Gelcer et al. 2010). For the case when only temperature data were available, it performed slightly better than the FAO 56 procedure (Table 3) but still yielded a poor result. These findings are consistent with those of Sentelhas et al. (2010) in Canada, Igbadun et al. (2006) in Tanzania, and Fooladmand and Haghighat (2007) in Iran. In contrast, the FAO 56 method was found to be superior in Bulgaria (Popova et al. 2006) and Bolivia (Garcia et al. 2004). Clearly the application of the FAO 56 method for wet páramo landscapes—as well as that of other more simple approaches—needs to be further investigated.

Conclusions

Our study provides a comprehensive analysis of how well $E_{To}$ can be calculated for the wet páramo of southern Ecuador if data for one or more meteorological variables are missing. For this landscape, high-quality meteorological data are rarely available, and for this reason it is especially crucial to understand to what extent $E_{To}$ calculations are dependent on such data. We found that all of the variables, wind speed is the least important and that excellent $E_{To}$ estimates can be made using the global average for wind speed. RH data are more important than wind speed, and the most important variable, according to our results, is $R_s$. This study demonstrates the importance of long-term collection of high-quality meteorological data, which will make it possible to develop new algorithms and calibrate the existing ones for calculating $E_{To}$ in the high-elevation Andes.

The study findings have not only improved our understanding of the accuracy of $E_{To}$ estimates when data are incomplete, but in particular we now know that without actual RH and $R_s$ data, $E_{To}$ will be poorly estimated. We can conclude, therefore, that for the wet páramo, more extensive climate monitoring is an urgent need, as is the development of alternative techniques for estimating values for missing variables.

Given the similarities between our results and those found by Sentelhas et al. (2010), who also studied the FAO 56 method in a cold, humid climate, we expect that these findings are transferable to other high-mountain regions with a cold, humid climate. However, we recognize that there is also a need for similar studies in different Andean ecosystems such as the dry páramo, the puna, and the altiplano.
ACKNOWLEDGMENTS

The authors are thankful to the Ecuadorian Secretary of Higher Education, Science and Technology and Innovation, the Research Office of the University of Cuenca, and Corporación Eléctrica del Ecuador CELEC EP-Hidropuente for funding this study (Grant No. PIC-11-715). We are grateful to the Ecuadorian Fulbright Commission for providing support for B.W. The authors acknowledge the staff of the Loma Larga project, Empresas de Telefonía, Agua Potable y Alcantarillado ETAPA-Parque Nacional Cajas, and the Comunidad de Chumbilin for their logistics support and facilities during data collection. Special thanks are due to the students and staff who contributed to the meteorological monitoring.

REFERENCES


Supplemental material

APPENDIX SI  The annual cycle of $R_a$ and its influence on $R_s$ estimates.

Found at DOI: http://dx.doi.org/10.1659/MRD-JOURNAL-D-15-0024.S1 (100KB PDF).